

COMPARISON OF VARIOUS RANS AND RANS-LES HYBRID MODELS ON A TRANSONIC DIFFUSER PROBLEM

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Abstract

The purpose of this article is to compare various CFD approaches for modeling turbulent flows, specifically the two-equation Reynolds-averaged Navier–Stokes (RANS) Menter’s Shear Stress Transport turbulence model (SST) and its various hybrids with large eddy simulation (LES): detached-eddy simulation (DES), delayed DES (DDES) and improved delayed DES (IDDES). We compare these RANS and hybrid RANS-LES methods on a diffuser problem with experimental data.

Keywords: CFD, finite volumes, RANS–LES, DES, DDES, IDDES, diffuser, transonic, Orion software.

1 Introduction

Turbulent flow pose significant challenges in the field of computational fluid dynamics. While direct numerical simulation (DNS) provides accurate results for all the spatial turbulent scales, the computational requirements are too high even for current hardware. Large eddy simulation (LES) provides acceptable alternative for DNS, but it also requires very fine meshes that often keep its computational costs still too high. To solve this problem, less demanding Reynolds-averaged Navier–Stokes Equations (RANS) methods can be utilized. However, while RANS models may be sufficient for some applications, they come with significant drawbacks, as they only resolve large turbulent length scales and have difficulties with computing unsteady and massively separated flows.

To bridge the gap between satisfying results of LES and computational demands of RANS models, various hybrid RANS–LES approaches have been proposed. One of the most prevalent hybrid methods, detached eddy simulation (DES), uses RANS model as a basis and switches to a subgrid-scale model in regions with sufficient local flow properties and computational grid resolution to allow LES formulation. As a consequence, the boundary layer is usually resolved by the RANS model and detached eddies by the LES model. This approach can produce reasonable results for larger eddies while retaining the advantages of RANS model and being completely non-zonal.

One of the main drawbacks of this method is its dependency on the computational grid resolution, which can lead to switching into the LES mode in areas where RANS mode should be used instead. In extreme cases, this can even lead to phenomena commonly referred to as Modeled Stress Depletion (MSD) and Grid Induced Separation (GIS) [1].

While DES models use simple binary switch between RANS and LES regions, more sophisticated approaches can be used instead. Delayed DES is one such model, thanks to the use of a more general blending function, eliminating the problems mentioned above. This method also has its own improved variant, simply called Improved delayed DES (IDDES), which introduces a wall-modeled LES (WMLES) functionality into DDES and further improves the blending function, which is now also dependent on the distance from the nearest wall.

The aim of this work is to compare finite volume methods for turbulence modeling, specifically the SST and its DES, DDES and IDDES models, on a case without separation, where the RANS model by itself should be sufficient. The reasoning for this is to evaluate the hybrid models on a type of flow that can play a significant role even in cases where massive flow separation is present, so we can determine the behavior of these methods before adopting them for other problems. For this purpose, we use our in-house implicit solver code Orion to compute a chosen diffuser problem with a turbulent boundary layer.

For our comparison, we chose the Sajben transonic converging-diverging diffuser validation test case used by NPARC (National Project for Application-oriented Research in CFD) Alliance (partnership between the NASA Glenn Research Center and the Arnold Engineering Development Center) published at [2]. This test case originates from the study [3] conducted to investigate off-design operating conditions at the inlet of supersonic aircraft engines. To validate our CFD methods, we will use the experimental data from the measurements presented in [4]. For our comparison, we chose one of the two interesting measured configurations described there, specifically the so-called “weak shock case”, which is a case with an unseparated turbulent boundary layer.

2 Mathematical model

2.1 Reynolds-averaged Navier–Stokes equations

In this paper, we use the Reynolds-Averaged Navier–Stokes (RANS) equations to describe the fluid motion. We assume the flow of an ideal compressible fluid. We will also ignore any body accelerations acting on our continuum, e.g. gravity.

After applying conventional Reynolds time-averaging and Favre averaging (density-weighted) to the Navier–Stokes equations, our RANS equations can be written using the averaged quantities as follows [5]:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0, \quad (1a)$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \operatorname{div}((\rho \mathbf{u}) \otimes \mathbf{u}) = \operatorname{div} \boldsymbol{\sigma} + \operatorname{div} \boldsymbol{\tau} - \operatorname{grad} p, \quad (1b)$$

$$\frac{\partial \rho E}{\partial t} + \operatorname{div}(\rho \mathbf{u} H) = \operatorname{div}(\boldsymbol{\tau} \mathbf{u}) - \operatorname{div} \mathbf{q} + \operatorname{div} \left(\boldsymbol{\sigma} \mathbf{u} + \left[\mu + \frac{\mu_T}{\sigma_k} \right] \operatorname{grad} k + \frac{c_p \mu_T}{\operatorname{Pr}_T} \operatorname{grad} T \right), \quad (1c)$$

where $\otimes$ denotes the outer product, ρ is the fluid density, t is time, $\mathbf{u}$ is the fluid velocity with u_1, u_2, u_3 as its components, p is the pressure, E and H (given by $H = E + \frac{p}{\rho}$) represent the specific energy and enthalpy, respectively, $\mathbf{q}$ denotes the heat-flux given by the Fourier’s law, T is the temperature, c_p is the heat capacity at constant pressure, Pr represents the Prandtl number and Pr_T the turbulent Prandtl number, μ is the dynamic viscosity, μ_T denotes the eddy viscosity, σ_k is a coefficient given by a turbulence model, $\boldsymbol{\sigma}$ is the viscous stress tensor and $\boldsymbol{\tau}$ represents the Reynolds stress tensor. We approximate the heat-flux as

$$\mathbf{q} \approx -\frac{c_p \mu}{\operatorname{Pr}} \operatorname{grad} T. \quad (2)$$

We also assume the following approximation of the viscous stress tensor (assuming $\mathbf{I}$ is a 3×3 identity matrix):

$$\boldsymbol{\sigma} \approx 2\mu \left(\mathbf{S} - \frac{1}{3} \operatorname{div} \mathbf{u} \mathbf{I} \right), \quad (3)$$

where $\mathbf{S}$ denotes the strain tensor, which has its components defined as

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (4)$$

Lastly, $\boldsymbol{\tau}$ is obtained by the Boussinesq approximation [6]

$$\boldsymbol{\tau} = \mu_T \left(\mathbf{S} - \frac{1}{3} \operatorname{div} \mathbf{u} \mathbf{I} \right) - \frac{2}{3} \rho k \mathbf{I}. \quad (5)$$

Here, we introduced the eddy viscosity μ_T , similarly to the dynamic viscosity in the stress tensor. The eddy viscosity needs to be obtained by a turbulence model.

To complete the system (1), we use the equation of state for the ideal gas and the turbulence model described in the next section.

2.2 Turbulence models

To obtain the eddy viscosity μ_T introduced by the Boussinesq hypothesis (5), we first formulate our two-equation turbulence models in conservation form, based on a variation of Menter's SST model from 2003 [7], with DES modification by Strelets [8]:

$$\frac{\partial \rho k}{\partial t} + \operatorname{div}(\rho k \mathbf{u}) = \operatorname{div}([\mu + \sigma_k \mu_T] \operatorname{grad} k) + P - \rho k \frac{\sqrt{k}}{L_T}, \quad (6a)$$

$$\frac{\partial \rho \omega}{\partial t} + \operatorname{div}(\rho \omega \mathbf{u}) = \operatorname{div}([\mu + \sigma_\omega \mu_T] \operatorname{grad} \omega) + \frac{C_\omega \rho}{\mu_T} P - \beta \rho \omega^2 + 2(1 - F_1) \frac{\rho \sigma_{\omega_2}}{\omega} \nabla \omega \nabla k, \quad (6b)$$

where ρk and $\rho \omega$ are our conservative variables, k is the turbulent kinetic energy, ω is the specific rate of dissipation, L_T is the model length scale (the only term different for each of the described methods), F_1 is the SST blending function (see [7] for details), P is the SST-limited production and the coefficients are $\sigma_k = 0.85$, $\sigma_\omega = 0.5$, $\sigma_{\omega_2} = 0.856$, $C_\omega = 0.533$. Let us consider the RANS and LES length scales:

$$l_{\text{RANS}} = \frac{\sqrt{k}}{\beta^* \omega}, \quad l_{\text{LES}} = C_{\text{DES}} \Delta, \quad (7)$$

where Δ is the maximum length of the cell's edges, $\beta^* = 0.09$ and

$$C_{\text{DES}} = (1 - F_1) C_{\text{DES}}^{k-\varepsilon} + F_1 C_{\text{DES}}^{k-\omega}, \quad (8)$$

$$C_{\text{DES}}^{k-\varepsilon} = 0.61, \quad C_{\text{DES}}^{k-\omega} = 0.78. \quad (9)$$

Now we can write the length scale of the RANS SST [7] model as

$$L_T^{(\text{SST})} = l_{\text{RANS}}. \quad (10)$$

DES model [8] uses the length scale

$$L_T^{(\text{DES})} = \min(l_{\text{RANS}}, l_{\text{LES}}). \quad (11)$$

This equation shows us the basic binary switching method between RANS and SGS LES models. Unfortunately, while the RANS method can be utilized everywhere, the length scale l_{LES} should not be chosen in certain regions, e.g. most areas in very close proximity to walls. However, if the computational grid contains small enough cells, the prescribed condition (11) may incorrectly direct the switch to LES mode in zones where RANS mode should be employed instead.

To address the aforementioned shortcomings of the basic DES method, a shielding function f_d , as presented in [1], can be introduced to formulate the DDES length scale:

$$L_T^{(\text{DDES})} = l_{\text{RANS}} - f_d \max(0, l_{\text{RANS}} - l_{\text{LES}}). \quad (12)$$

Since the values of the shielding function give us a non-binary criterion, the areas fully in RANS mode smoothly transition into the LES regions. This method is more conservative with full LES mode to prevent unwanted switching, while still leaving sufficient tolerance for resolving phenomena where RANS method is undesirable (e.g. detached eddies) with the LES model.

We can also further extend the previous formulation of the DDES method by adding the improved near-wall-modelling capabilities known from the WMLES methods. Thus, we formulate the IDDES length scale by introducing the empiric blending function $\tilde{f}_d$ and the elevating function f_e described in [1]:

$$L_T^{(\text{IDDES})} = \tilde{f}_d (1 + f_e) l_{\text{RANS}} + (1 - \tilde{f}_d) \hat{l}_{\text{LES}}, \quad (13)$$

where we modified the grid spacing with the distance from the nearest wall d_w :

$$\hat{l}_{\text{LES}} = C_{\text{DES}} \hat{\Delta}, \quad (14)$$

$$\hat{\Delta} = \min\{C_w \max[d_w, \Delta], \Delta\}, \quad C_w = 0.15. \quad (15)$$

Finally, after resolving the system (6) with the model of our choice, we obtain the k and ω needed to compute the eddy viscosity:

$$\mu_T = \frac{a_1 \rho k}{\max(a_1 \omega, F_2 S)}, \quad (16)$$

where $S = \sqrt{2S^2}$ and F_2 is the SST damping function described in [7].

2.3 Boundary conditions

As stated earlier, we impose the following boundary conditions to our computational domain: no-slip wall, symmetry, inlet and outlet. For the inlet condition, we need to specify the total pressure p_0 , total density ρ_0 as well as the kinetic turbulent energy k_∞ and specific dissipation rate ω_∞ of the incoming stream. For our diffuser problem, the inflow is strictly horizontal, so we do not need to specify any deviations of the angle of attack. On the outlet, the static pressure p_2 is prescribed.

3 Numerical methods

The previous section described two systems of differential equations that we need to solve: the RANS system (1) and the system for the chosen turbulence model (6). To do that, we use our in-house developed code Orion. In our approach, we solve these two systems of equations separately – first, we start with the RANS system, then we solve the system for the turbulence model to finish the time step.

The conservative variables in the first system are ρ , ρu_1 , ρu_2 , ρu_3 and ρE , while the conservative variables of the second system are ρk and $\rho \omega$. If we choose any of the given systems and arrange its variables in a vector $\mathbf{W}$, the terms not including the operator of divergence on the right side in a source term vector $\mathbf{Q}$ and we keep all the remaining terms on their respective sides of the equation, then the systems (1) and (6) can be both written in the following vector form:

$$\frac{\partial \mathbf{W}}{\partial t} + \text{div } \mathbf{F}_C(\mathbf{W}) = \text{div } \mathbf{F}_D(\mathbf{W}) + \mathbf{Q}(\mathbf{W}), \quad (17)$$

where $\mathbf{F}_C(\mathbf{W})$ is the vector of the convective fluxes, while the dissipative fluxes from the right-hand side are denoted as $\mathbf{F}_D(\mathbf{W})$.

To solve any of the variants of (17), we split the computational domain into cells with values of the conservative variables $\mathbf{W}_i$ for a cell indexed as i . In residual form, we can rewrite our spatially discretized system for each cell as

$$\left(\frac{\partial \mathbf{W}}{\partial t} \right)_i = -R_i(\mathbf{W}). \quad (18)$$

To calculate the residual on the right-hand side, we sum all the fluxes through every face of the cell i and also add the source term in the given cell. In our finite volume approach, we replace the convective fluxes in (1) with numerical fluxes from the HLLC Riemann solver described by Toro in [9]. For convective flux evaluation, we adopt a piecewise linear reconstruction technique and utilize the Barth-Jespersen limiter. For this part, we use first order analytical Jacobians.

To compute the gradients for the diffusive fluxes $\mathbf{F}_D$ evaluated in the face centers, we take values in points of “diamond cell” described in [10]. To get the values in the mesh nodes, we use the least square method. The gradient of a function ξ on a face f with points from its diamond cell indexed as r (so ξ has values ξ_{fr} at these points) will then be approximated as $\nabla \xi_f = \sum_r \alpha_r \xi_{fr}$, where α_r are the weights obtained according to [10].

For our temporal discretization we take the following implicit form of the second order backward differentiation formula (BDF2):

$$\frac{3\mathbf{W}_i^{n+1} - 4\mathbf{W}_i^n + \mathbf{W}_i^{n-1}}{2\Delta t} + \mathbf{R}_i(\mathbf{W}^{n+1}) = 0, \quad (19)$$

where the superindices indicate the time step from which the value of $\mathbf{W}_k$ comes from and Δt is the size of our time step. We can now write the incremental form for our conservative variable each time step: $\Delta \mathbf{W}_i^{n+1} = \mathbf{W}_i^{n+1} - \mathbf{W}_i^n$. The new unknown $\Delta \mathbf{W}^{n+1}$ can be obtained from the linearized system

$$\frac{3}{2} \Delta \mathbf{W}_i^{n+1} + \Delta t \sum_j \frac{\partial \mathbf{R}_i(\mathbf{W}^n)}{\partial \mathbf{W}_j} \Delta \mathbf{W}_j^{n+1} = -\Delta t \mathbf{R}_i(\mathbf{W}^n) + \frac{1}{2} \Delta \mathbf{W}_i^n, \quad (20)$$

where the indices j denote values in cells having a common face with the cell i .

The previous approximation gives us only first order of accuracy. To achieve the second order of accuracy, we utilize the dual time-stepping technique. For this approach, we will have to rewrite the spatial discretization (18) with the dual time τ :

$$\left(\frac{\partial \mathbf{W}}{\partial \tau}\right)_i + \left(\frac{\partial \mathbf{W}}{\partial t}\right)_i = -R_i(\mathbf{W}). \quad (21)$$

To discretize in the dual time, we reformulate the equation (19) and use the first order BDF to obtain

$$\frac{\mathbf{W}_i^{n+1,\ell+1} - \mathbf{W}_i^{n+1,\ell}}{\Delta \tau} + \frac{3\mathbf{W}_i^{n+1,\ell+1} - 4\mathbf{W}_i^n + \mathbf{W}_i^{n-1}}{2\Delta t} + \mathbf{R}_i(\mathbf{W}^{n+1,\ell+1}) = 0, \quad (22)$$

where ℓ denotes the current time step in the dual time (with the first step being $\mathbf{W}^n = \mathbf{W}^{n+1,1}$). The idea behind this approach is to iterate over the dual time steps until we get a value of $\mathbf{W}^{n+1,\ell}$ that is sufficiently close to the value of $\mathbf{W}^{n+1}$, which is considered stationary in the dual time formulation, so we assert $\mathbf{W}^{n+1,\ell_{\max}} = \mathbf{W}^{n+1}$ after our last iteration in the dual time $\ell_{\max}$ for the given (physical) time step $n + 1$. This implicit system is then linearized and rewritten in incremental form in exactly the same way as (20).

Lastly, to improve performance, we use local time-stepping in each dual time iterative process. The whole system for the $\Delta \mathbf{W}^{n+1,\ell+1}$ is solved by using the generalized minimal residual method.

4 Test case description

Our computational grid is three-dimensional and the diffuser is oriented horizontally. The shape of the diffuser is two-dimensional with no curvature from the front to the back, so the vertical cross-section of our grid is rectangular. The width of the throat is $h = 0.04399788$ m.

We prescribe symmetry on the front and back planes, while the top and bottom boundaries are adiabatic no-slip walls. Lastly, we specify the inlet boundary condition on the left side and the outlet boundary condition on the right side. Our computational grid contains 149 cells in the longitudinal direction, 231 in the vertical direction (distributed unevenly for better resolution alongside both walls) and 8 in the transverse direction, for a total of 275,352 cells.

The thickness of the first cell from the wall is always $1.4 \cdot 10^{-6}$ m (the dimensionless wall distance y^+ lies between 0.29 and 0.97), then it increases by 10 % until it reaches 0.005 m from the wall, from where the thickness is constant. The grid is evenly spaced in the other two directions, being $12.68h$ long in the longitudinal direction and measuring 0.018 m from front to back.

Values on the boundaries are $p_0 = 135$ kPa, $\rho_0 = 1.7$ kg $\cdot$ m $^{-3}$, $k_\infty = 9.7$ m 2 $\cdot$ s $^{-2}$, $\omega_\infty = 6,771$ s $^{-1}$ and $p_2 = 97.2$ kPa. Next, we prescribe the viscosity value $\mu = 1.82 \cdot 10^{-5}$, $\text{Pr} = 0.72$ and $\text{Pr}_T = 0.9$. The inlet Reynolds number is around 775,000.

5 Results

The flow characteristics are given by the converging nozzle combined with the low outlet static pressure p_2 , creating a supersonic flow and a shock wave that reduces the velocity back to subsonic levels (see Fig. 1 for the Mach number distribution by the DDES model).

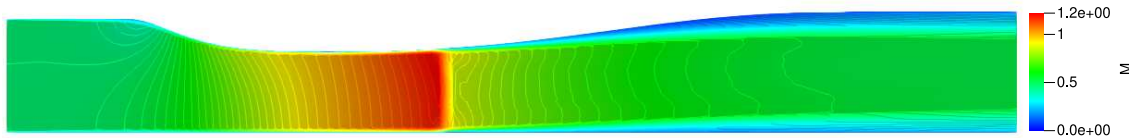


Figure 1: Mach number obtained by the DDES model.

As discussed earlier, the main difference between the presented turbulence models is their switching between RANS and LES mode. The simple binary DES switch shows significant differences from its more advanced counterparts DDES and IDDES, which, on the contrary, have quite

similar blending functions f_d and $\tilde{f}_d$, respectively, as apparent from Fig. 2. However, it is the overall difference between the switches described by (12) and (13) that makes these methods very different.

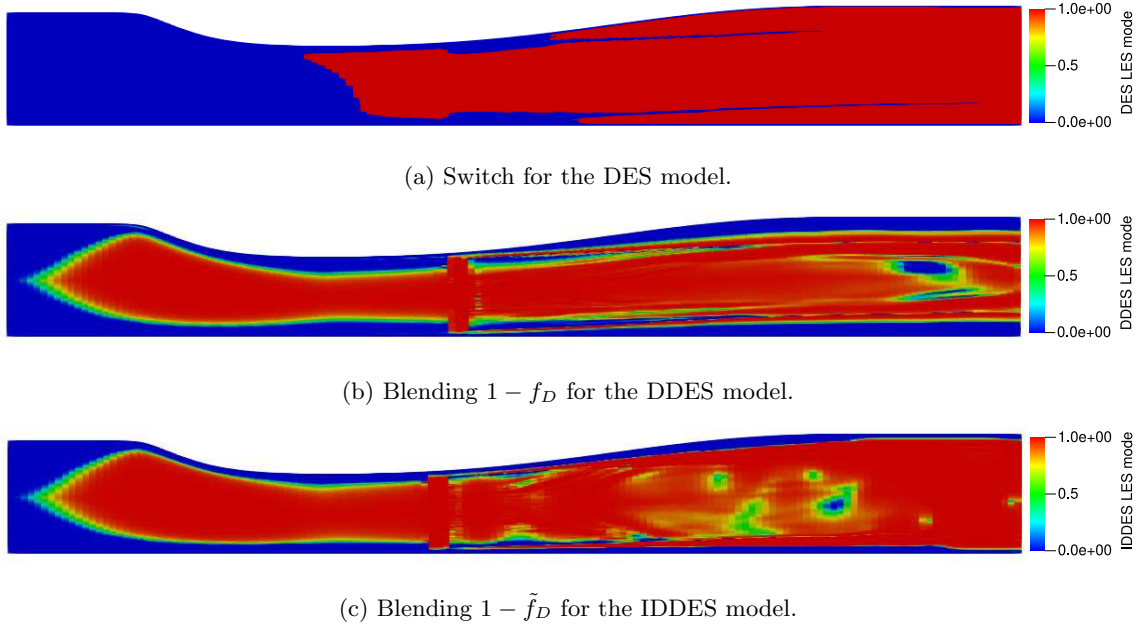
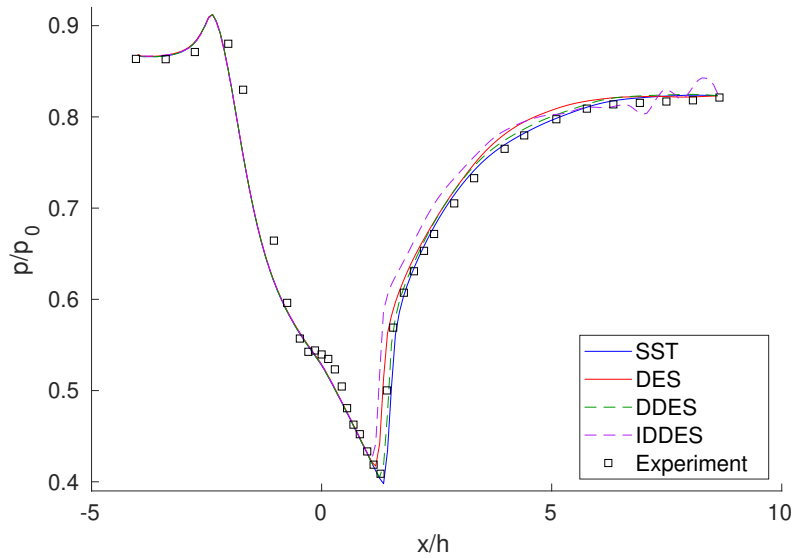


Figure 2: Different variants of RANS-LES switching functions – values of 0 and 1 indicate pure RANS or LES mode, respectively.

It is worth noting that the DES model uses its fully-LES mode much closer to the wall than the other models. On the other hand, the DDES and IDDES blending functions switch to LES mode much closer to the inlet. The switching in the shock wave region is also different – all models switch to the SGS method before the shock wave, but only DDES and IDDES do it on its front as well.



One of the particularly interesting parameters of the models is how well they predict the position

of the shock wave itself. This can be compared by looking at the static pressure (i.e., p/p_0) profile along the top wall shown in Fig. 3 (and the results on the bottom are very similar). We can see that the RANS-only SST model slightly overpredicts the position of the shock wave, while the DDES method exactly matches the experiment around the front of the shock wave. The simple DES approach, according to the previously mentioned Fig. 2a, seems to be too generous with the switching to LES mode, as it visibly underpredicts the shock wave position. The IDDES model fared even worse and the results also show some oscillations at the outflow. This is most likely due to the fact that for this approach, the LES regions were visibly closer to the walls as well as the resulting skin friction coefficient along the walls reached considerably higher values than for any other tested method.

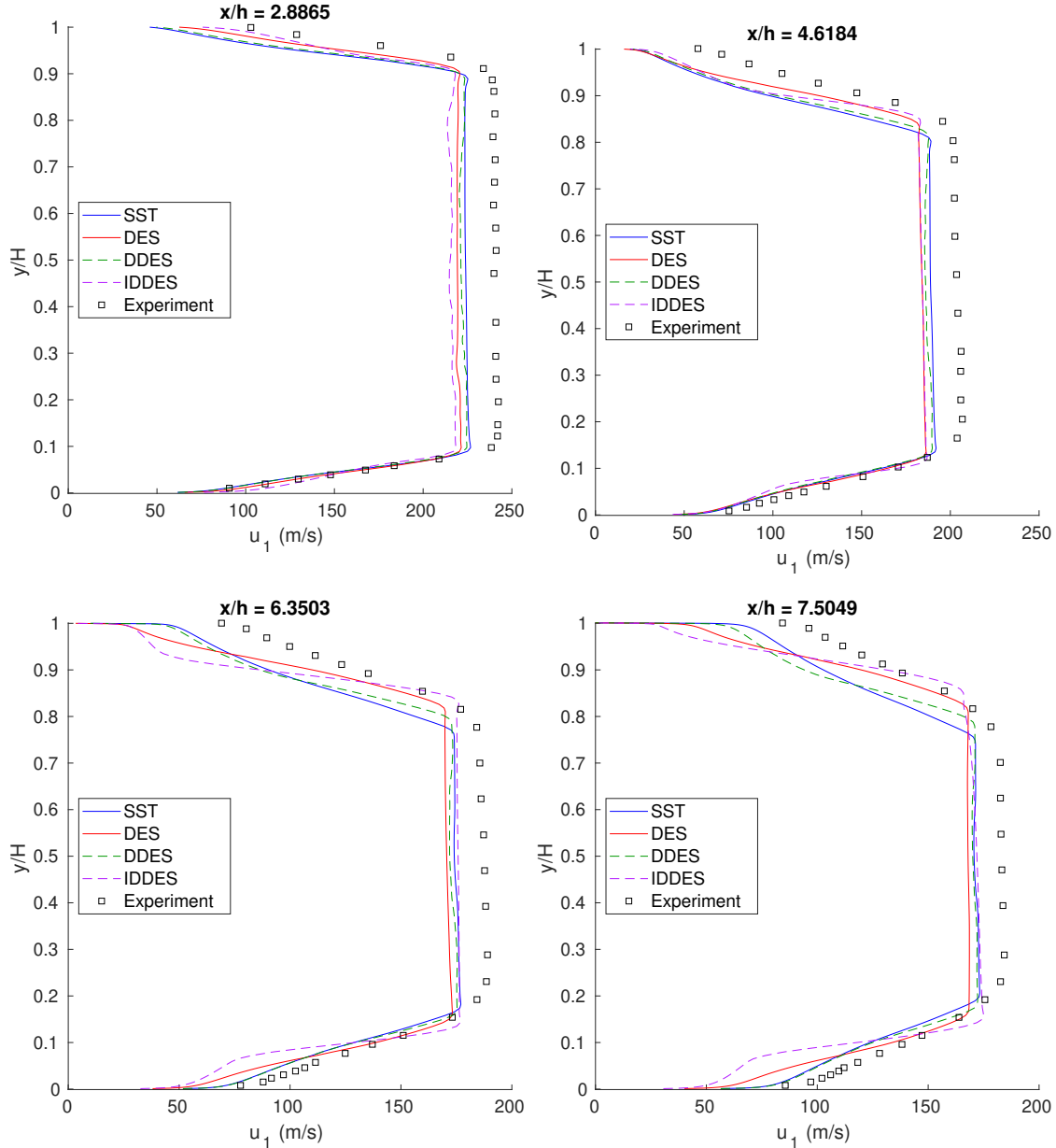


Figure 4: Results for the longitudinal velocities at different cross-sections, H being the height of the channel at the given location.

For the last metric to compare these RANS-LES hybrids, we present the velocity profiles shown in Fig. 4. The computed velocities are overall lower than those obtained experimentally. While the

SST and DDES models behave very similarly with no clear winner, the DES method seems to get better closer to the walls while underpredicting velocities in the middle of the channel. The IDDES approach differs the most, significantly struggling along the walls. Again, this is most likely due to the high calculated values of the skin friction coefficient. The velocity in the middle of the channel is also the lowest below the experimental value close to the shock wave, but the profile improves substantially with the increasing distance from the nozzle. In fact, the velocity profile away from the wall is there the closest to the experimental data of all the compared methods.

6 Conclusion

In this paper, we presented a comparison of some hybrid RANS-LES methods on the weak-shock validation case of the Sajben transonic diffuser. The most straightforward DES method seems to provide acceptable improvement in some areas, but its simplicity can sometimes lead to overly generous switching to LES mode. The most complex of the presented models, IDDES, has the opposite problem – it can be too complicated for its own good, as demonstrated by the comparison with the experimental data or even RANS approach. The high value of skin friction obtained by the IDDES requires some further investigation, possibly even recalibration of its coefficients. Finally, we were most successful with the DDES model, which seems to blend RANS and LES modes quite well, as confirmed by its results, which are usually closest to the experimental data.

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