

TWO SHOCK WAVES SELF-INDUCED OSCILLATIONS OVER NACA 0012 AIRFOIL AT LOW ANGLES OF ATTACK

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Abstract

The self-oscillations mode of two shock waves in a transonic turbulent flow around a NACA 0012 airfoil at low angles of attack is studied numerically. The physical features of the occurrence of self-oscillations are discussed, as well as their difference from other stationary and non-stationary modes of the transonic flow around the NACA 0012 airfoil. An updated map of the transonic turbulent flow regimes around the NACA 0012 airfoil on the plane free-stream Mach number angle of attack is presented.

Keywords: transonic flow, airfoil, shock waves, self-induced oscillations, turbulence, Navier-Stokes equations, numerical solution, flow modes

1 Introduction

The transonic speed range is the base for passenger aircraft flights. Symmetric airfoils similar to NACA 0012 form the basis for the design of vertical controls that are at zero angle of attack in cruise flight. The occurrence of self-oscillations of shock waves can lead to buffeting of the tail elements, which, in turn, leads to the destruction of the controls, or, at least, to the loss of controllability of the aircraft. In spite of the fact that investigations into the abovementioned region have been carried out for more than half a century, the phenomenon of the viscous-inviscid interaction that leads to a different structure of flow field remains little studied [1-6].

To date, understanding the general physical principles of transonic flow over airfoils has been achieved, including the processes of the appearance of local supersonic zones, shocks, and rarefaction waves, and development of the boundary layer and its separation. At the same time, it is necessary to carry out physical or numerical experiments to obtain quantitative data on the concrete flow structure, aerodynamic characteristics at a specified airfoil shape, angle of attack, and free-stream Reynolds and Mach numbers.

In paper [6] a systematic study was carried out in a wide range of Mach numbers and angles of attack of a transonic turbulent flow around a NACA 0012 airfoil, identifying zones of individual regimes on the plane "Mach number - angle of attack", including self-oscillations of the shock wave, based on numerical simulation using Navier-Stokes equations. The presented study is a continuation of investigations [6] for the mode of self-oscillations of two shock waves (fig. 1).

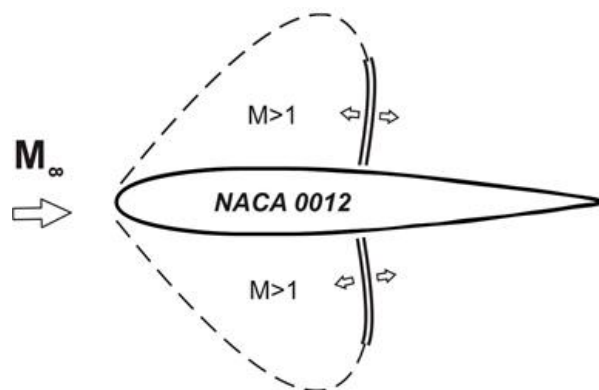


Figure 1: Scheme of transonic flow with two shock waves self-induced oscillations

2 Problem formulation

Transonic flows near the symmetric airfoil NACA 0012 are investigated at small angles of attack $\alpha = 0^\circ \div 2^\circ$ at Reynolds number $Re = 3.91 \cdot 10^6$. The range of Mach numbers was $M_\infty = 0.84 \div 0.87$.

Numerical simulation of unsteady transonic turbulent flow over airfoils was carried out using two-dimensional URANS equations written in dimensionless form for arbitrary curvilinear coordinates [6-8].

$$\frac{\partial \hat{\mathbf{q}}}{\partial t} + \frac{\partial \hat{\mathbf{E}}}{\partial \xi} + \frac{\partial \hat{\mathbf{F}}}{\partial \eta} = \frac{1}{Re} \left(\frac{\partial \hat{\mathbf{E}}_v}{\partial \xi} + \frac{\partial \hat{\mathbf{F}}_v}{\partial \eta} \right), \quad (1)$$

$$\hat{\mathbf{q}} = \frac{\mathbf{q}}{J}, \quad \mathbf{q} = (\rho, \rho u, \rho v, e)^T, \quad (2)$$

$$\hat{\mathbf{E}} = \frac{1}{J} \begin{bmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ U(e + p) \end{bmatrix}, \quad \hat{\mathbf{F}} = \frac{1}{J} \begin{bmatrix} \rho V \\ \rho u V + \eta_x p \\ \rho v V + \eta_y p \\ V(e + p) \end{bmatrix}, \quad (3)$$

$$\hat{\mathbf{E}}_v = \frac{1}{J} \begin{bmatrix} 0 \\ \xi_x \tau_{xx} + \xi_y \tau_{xy} \\ \xi_x \tau_{yx} + \xi_y \tau_{yy} \\ \xi_x \beta_x + \xi_y \beta_y \end{bmatrix}, \quad \hat{\mathbf{F}}_v = \frac{1}{J} \begin{bmatrix} 0 \\ \eta_x \tau_{xx} + \eta_y \tau_{xy} \\ \eta_x \tau_{yx} + \eta_y \tau_{yy} \\ \eta_x \beta_x + \eta_y \beta_y \end{bmatrix}, \quad (4)$$

$$U = \xi_x u + \xi_y v, \quad V = \eta_x u + \eta_y v, \quad (5)$$

$$\tau_{xx} = \mu_T \left[4(\xi_x u_\xi + \eta_x u_\eta) - 2(\xi_y v_\xi + \eta_y v_\eta) \right] / 3, \quad (6)$$

$$\tau_{yy} = \mu_T \left[4(\xi_y v_\xi + \eta_y v_\eta) - 2(\xi_x u_\xi + \eta_x u_\eta) \right] / 3, \quad (7)$$

$$\tau_{xy} = \tau_{yx} = \mu_T \left[(\xi_y u_\xi + \eta_y u_\eta) + (\xi_x v_\xi + \eta_x v_\eta) \right], \quad (8)$$

$$\beta_x = u \tau_{xx} + v \tau_{xy} + \frac{1}{(\gamma - 1)} \left(\frac{\mu_t}{Pr_t} + \frac{\mu_l}{Pr_l} \right) \cdot \left(\xi_x \frac{\partial a^2}{\partial \xi} + \eta_x \frac{\partial a^2}{\partial \eta} \right), \quad (9)$$

$$\beta_y = u \tau_{yx} + v \tau_{yy} + \frac{1}{(\gamma - 1)} \left(\frac{\mu_t}{Pr_t} + \frac{\mu_l}{Pr_l} \right) \cdot \left(\xi_y \frac{\partial a^2}{\partial \xi} + \eta_y \frac{\partial a^2}{\partial \eta} \right). \quad (10)$$

The following designations are accepted in these equations: u and v are the velocity vector components in the x and y directions; ρ , p , and e are the density, pressure, and total energy of gas in the unit volume; a is the speed of sound; γ is the ratio of specific heats; μ and μ_t are the dynamic coefficients of molecular and turbulent viscosity; $\mu_T = \mu + \mu_t$ is the dynamic coefficient of the effective viscosity; Re is the Reynolds number; $Pr = 0.72$ and $Pr_t = 0.9$ are the Prandtl numbers for the laminar and turbulent flow regimes, correspondingly; and ξ_x , ξ_y , η_x , η_y and J are the metric coefficients and Jacobian of the transformation of coordinates.

The set of governing equations is supplemented with the state equation of perfect gas. The dynamic coefficient of molecular viscosity μ is defined from the power dependence on temperature T $\mu/\mu_\infty = (T/T_\infty)^{0.76} = (a^2/a_\infty^2)^{0.76}$. To determine the coefficient of turbulent viscosity the one-parameter differential Spalart–Allmaras model [9] is used with transformation for curvilinear coordinates [6].

The non-slipping condition $u=v=0$ and heat-exchange-absence condition $\partial T/\partial n=0$ are set on the airfoil surface. On the outer boundary of the computational domain, the values of the flow parameters are determined on the basis of Riemann invariants [7, 8]. The value of operating variable $\tilde{v}$ of Spalart–Allmaras model on the solid surface is set equal to zero, and $\tilde{v}=0.1$ on the inlet outer boundary. Conditions without the gradient for $\tilde{v}$ are set on the outlet boundary.

3 Numerical algorithm

Discretization of the initial set of the equations is fulfilled using the finite-volume method for curvilinear coordinates [6, 8]. In this method, a control volume in the form of a rectangle with edges of lengths $\Delta \xi$

and $\Delta\eta$ – the interfaces of which are designated by half-integer indices – is set in the space of the variables $\{\xi, \eta\}$ around each mesh node.

A discrete analog of governing equations with second-order accuracy in time takes the following form

$$\frac{3\Delta\hat{\mathbf{q}}^n - \Delta\hat{\mathbf{q}}^{n-1}}{2\Delta t} + \hat{\mathbf{R}}^{n+1} = 0, \quad (11)$$

where n is a number of the time layer, $\Delta\hat{\mathbf{q}}^n = \hat{\mathbf{q}}^{n+1} - \hat{\mathbf{q}}^n$; and

$$\hat{\mathbf{R}}^{n+1} = \frac{\hat{\mathbf{E}}_{i+1/2}^{n+1} - \hat{\mathbf{E}}_{i-1/2}^{n+1}}{\Delta\xi} + \frac{\hat{\mathbf{F}}_{j+1/2}^{n+1} - \hat{\mathbf{F}}_{j-1/2}^{n+1}}{\Delta\eta} - \frac{1}{\text{Re}} \left(\frac{\hat{\mathbf{E}}_{v,i+1/2}^{n+1} - \hat{\mathbf{E}}_{v,i-1/2}^{n+1}}{\Delta\xi} + \frac{\hat{\mathbf{F}}_{v,j+1/2}^{n+1} - \hat{\mathbf{F}}_{v,j-1/2}^{n+1}}{\Delta\eta} \right). \quad (12)$$

To calculate the convective components, the Roe scheme is used [10], according to which the fluxes through the interface of cell $i + 1/2$ of the control volume are defined as follows:

$$\hat{\mathbf{E}}_{i+1/2} = \frac{1}{2} \left[\hat{\mathbf{E}}(\mathbf{q}_L) + \hat{\mathbf{E}}(\mathbf{q}_R) - |\tilde{\mathbf{A}}| \cdot (\mathbf{q}_R - \mathbf{q}_L) \right], \quad (13)$$

where $\mathbf{q}_L$ and $\mathbf{q}_R$ are the parameters of the fluxes from the left and right of the face, correspondingly. If $\mathbf{q}_L = \mathbf{q}_i$ and $\mathbf{q}_R = \mathbf{q}_{i+1}$, then the given scheme is first-order accurate. To provide second-order accuracy in space a symmetric flux limiter is applied

$$\mathbf{q}_L = \mathbf{q}_i + \psi(\Delta\mathbf{q}_{i-1/2}, \Delta\mathbf{q}_{i+3/2}), \quad (14)$$

$$\mathbf{q}_R = \mathbf{q}_{i+1} - \psi(\Delta\mathbf{q}_{i-1/2}, \Delta\mathbf{q}_{i+3/2}), \quad (14)$$

where ψ is a flux limiter; i.e., a function of the parameter differences at neighboring points $\Delta\mathbf{q}_{i-1/2} = \mathbf{q}_i - \mathbf{q}_{i-1}$, etc. In this work, the following symmetric limiter, suggested by Jameson [11] is used:

$$\psi(a, b) = \frac{1}{4}(a + b) \left[1 - \left(\frac{a - b}{|a| + |b| + \varepsilon} \right)^2 \right], \quad \varepsilon = 10^{-3}. \quad (16)$$

The choice of this flux limiter is governed by the fact that it was developed especially for flows with strong discontinuities forming on shocks.

To protect the conservation laws the metric coefficients on face $i + 1/2$ of the control volume are calculated using the following formula [6, 8]

$$\left(\frac{\xi_x}{J} \right)_{i+1/2} = \frac{1}{2} \left[\left(\frac{\xi_x}{J} \right)_i + \left(\frac{\xi_x}{J} \right)_{i+1} \right]. \quad (17)$$

The convective fluxes through other faces of the control volume are similarly calculated. The viscous terms in equations (12) are approximated by three-point stencil with second order accuracy.

While constructing the implicit algorithm, residual vector is linearized with respect to time layer n using the Taylor series of second-order of accuracy as follows

$$\hat{\mathbf{R}}^{n+1} = \hat{\mathbf{R}}^n + \left(\frac{\partial \hat{\mathbf{R}}}{\partial \hat{\mathbf{q}}} \right)^n \Delta\hat{\mathbf{q}}^n + O(\Delta\hat{\mathbf{q}}^n)^2. \quad (18)$$

After substitution in relations (11), the implicit operator appears as follows

$$\left(\mathbf{I} + \frac{2}{3} \Delta t \frac{\partial \hat{\mathbf{R}}}{\partial \hat{\mathbf{q}}} \right)^n \Delta\hat{\mathbf{q}}^n = \frac{1}{3} \Delta\hat{\mathbf{q}}^{n-1} - \frac{2}{3} \Delta t \hat{\mathbf{R}}^n, \quad (19)$$

where $\mathbf{I}$ is the unit matrix. The block-matrix set of algebraic equations (19) is solved by the Gauss–Seidel iteration algorithm.

The numerical algorithm introduced above is implemented in the authors CFD codes, which are developing in Institute of Transport Systems and Technologies of National Academy of Sciences of Ukraine [6, 12, 13]. No commercial CFD codes were used in the presented paper.

Testing of the numerical algorithm was carried out on several problems, which reproduced various regimes of flow over an airfoil. The calculations were carried out on type “O” meshes with number of nodes from 450x200 to 600x250. Clustering of the mesh nodes near the solid surface was chosen in such a way to provide a value of $\Delta y^+ < 0.7$ in the zone of viscous sublayer in turbulent boundary layer. The distance to the outer boundary was set equal to 20–25 airfoil chord lengths.

The steady subsonic and transonic flows over the NACA 0012, NACA 4412 RAE 2822 airfoils were simulated in comparison with the experimental data.

The results of the numerical algorithm testing for the regime of self-oscillation shock on the leeward side of the BGK No.1 airfoil are given in Fig. 2. The parameters of the free flow in accordance with the

experimental data [3, 4] were $M_\infty = 0.71$, $\alpha = 6.97^\circ$, and $Re = 2 \cdot 10^7$. In the considered case, the shock wave periodically oscillated in the zone, which covered approximately 42% of chord length. Fig. 2a and 2b show the shock waves in the extreme left and extreme right positions at the moments of time related to the non-dimensional oscillation period $T = 1/Sh$, where Sh is the Strouhal number. In the oscillation process, the shape and intensity of the shock wave and, correspondingly, the structure of separated flow changed. The frequency characteristic $k = \pi \cdot Sh$ was $k = 0.253$ in the calculation and $k = 0.25$ in the experiment [3, 4].

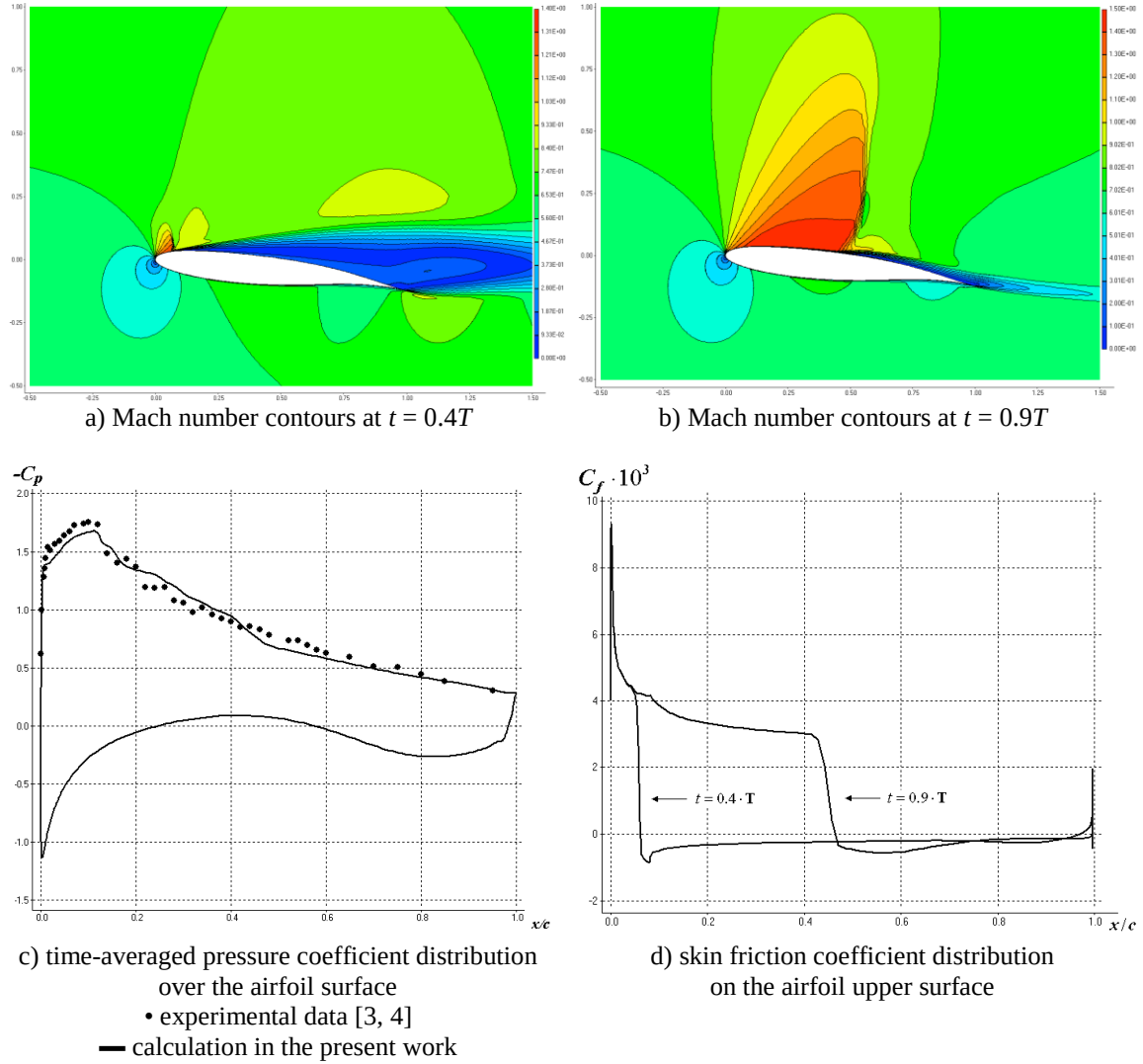


Figure 2 : Results of the calculation of the transonic flow with self-oscillation shock over the BGK No.1 airfoil at $M_\infty = 0.71$, $\alpha = 6.97^\circ$, $Re = 2.0 \cdot 10^7$

4 Numerical results

The series of computational experiments presented below was carried out similarly to test calculations on type “O” mesh with 600x500 nodes. Value Δy^+ near the solid surface was less than 0.4.

Fig. 3 shows a half-cycle of two shock waves motion near the NACA 0012 airfoil. Fig. 3a ($t = 0$) corresponds to the maximum value of the lifting force at $M_\infty = 0.855$, $\alpha = 0^\circ$. Different positions of shock waves are clearly visible, as well as fluctuations in the airfoil wake, i.e. the trailing edge exit angle is not fixed. Pressure fluctuations in the wake occur across the wake. Pressure changes behind the trailing edge also affect upstream and lead to asymmetric fluctuations in the distribution of shock waves.

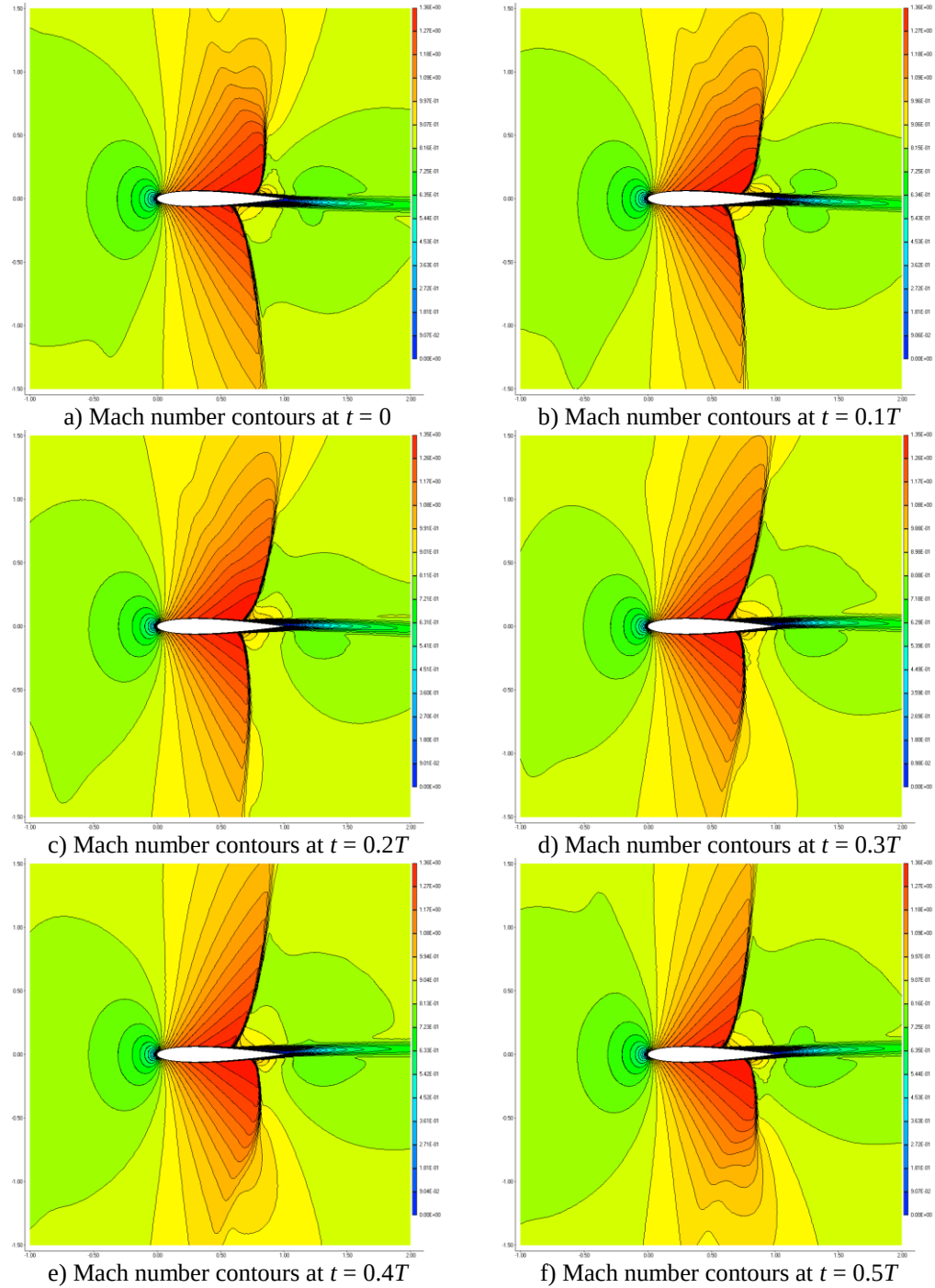


Figure 3: Distributions of local Mach numbers at $M_\infty = 0.855$, $\alpha = 0^\circ$ at different moments of shock waves oscillations

Processing the results of computational experiments made it possible to obtain dependences for the integral aerodynamic coefficients, which are presented in fig. 4-6.

Fig. 4 shows the evolution of the lift and drag coefficients at $M_\infty = 0.855$, $\alpha = 0^\circ$. The maximum value C_L was 0.1634. Non-dimensional period of oscillations $T = 1/Sh = 10.8$. The drag coefficients average value was $C_D = 0.061$ with fluctuations less 1%.

A full period of drag coefficient equals to a half-period in lift coefficient fluctuations due to the symmetry of the streamlined airfoil. The maximum value of C_D corresponds to the maximum or minimum of C_L . The minimum C_D corresponds to the values $C_L \approx 0$. Those, the maximum and minimum of the drag coefficient are not antiphase oscillations, but refer to different time moments of oscillations.

The effect of the free stream Mach number on the oscillations amplitude of the lift coefficient is shown in Fig.5. The calculations were carried out with a step of $M_\infty = 0.001$. The beginning of the zone of self-oscillations by $M_\infty = 0.844$, the end by $M_\infty = 0.866$. The maximum value C_L was equal to 0.1666 at $M_\infty = 0.866$. The non-monotonicity of the dependences obtained is related to the structure of the separated flow during oscillations. A change in the structure of the separated flow affects the non-dimensional oscillation period: at $M_\infty = 0.845$ $T = 15.1$ and at $M_\infty = 0.865$ $T = 9.53$.

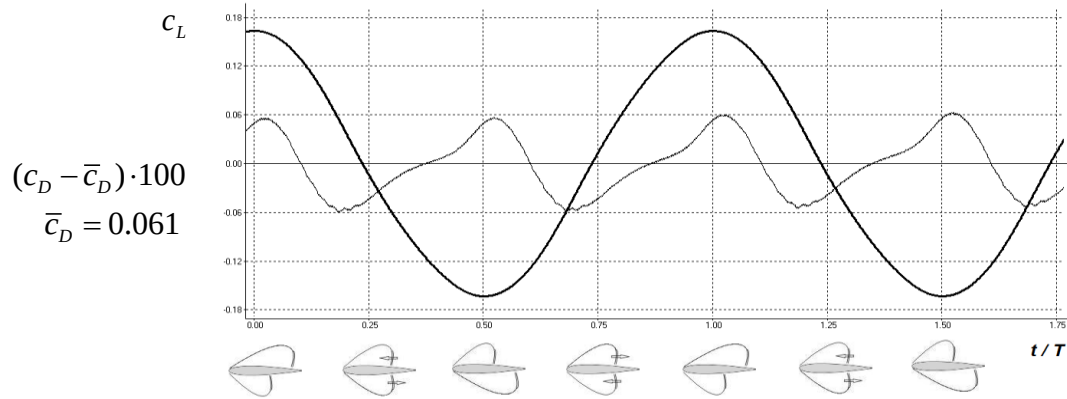


Figure 4: Time evolution of lift coefficient C_L (—) and drag coefficient C_D (---) at $M_\infty = 0.855$, $\alpha = 0^\circ$, referred to non-dimensional oscillation period $T = 10.8$

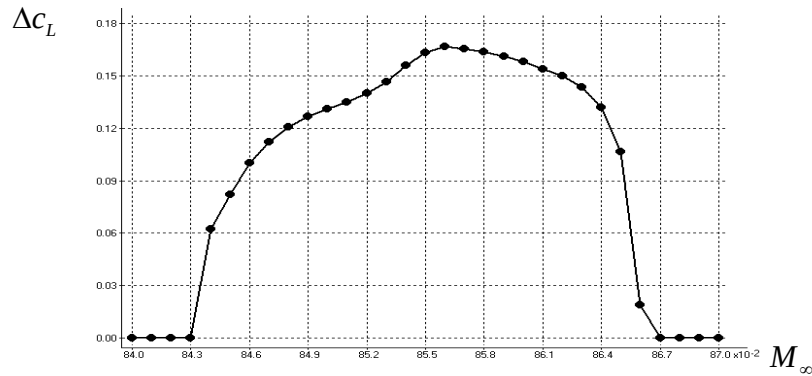


Figure 5: Influence of the Mach number of the incoming flow on the amplitude of lift coefficient fluctuations at $M_\infty = 0.84 \div 0.87$, $\alpha = 0^\circ$

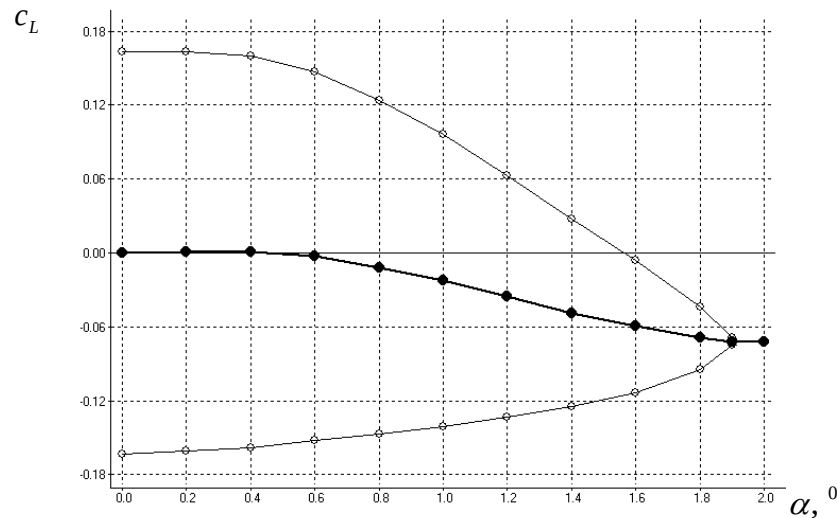


Figure 6: Influence of the angle of attack on the amplitude and the average value of the lift coefficient at $M_\infty = 0.855$

The angle of attack introduces asymmetry into the flow pattern, so it was interesting to study the effect of the angle of attack on the amplitude of oscillations and the average value of the lift coefficient at a fixed value of $M_\infty = 0.855$ (fig.6). The calculations were carried out with a step of $\alpha = 0.2^\circ$.

At very small angles of attack ($\alpha = 0.2^\circ$, $\alpha = 0.4^\circ$), the changes were insignificant - the amplitude of the oscillations practically did not change. Starting from the angle $\alpha = 0.4^\circ$ average value of the lift coefficient became negative, the oscillation amplitude decreased. This is explained by the fact that, due to the asymmetry of the flow, the average position of shock waves on the upper and lower surfaces is different. In other words, the shocks "spread". This "complicates" the air flow from the lower surface to the upper (and vice versa) around the trailing edge. The actual end of the self-oscillation mode was fixed at $\alpha = 1.9^\circ$. At $\alpha = 2.0^\circ$, a completely stationary flow around the NACA 0012 airfoil was obtained.

As was established earlier, [3-6] the physical feature of oscillations in the mode with one shock wave (mode A1) is the formation of a developed separation zone, which extends to the trailing edge. Growth of the separated flow zone near the trailing edge leads to an increase in wake pressure and displacement of the shock wave downstream along with the separation point. The separation zone splits into two: this is the separation caused by the shock wave and the separation near the trailing edge. In the extreme right position of the shock wave, only one separation zone from the shock wave remains, and the flow near the trailing edge on lower side is attached. Pressure pulsations occur in the wake downstream and upstream.

The existence of a non-stationary mode of self-oscillations of one shock wave (mode A1) is limited by the Mach number to stationary modes (fig. 7). With a decrease in the Mach number at a fixed angle of attack, a classical regime arises with a weak shock wave and a small flow separation zone near this shock wave. As the Mach number increases, the shock wave forms the well-known "lambda foot" configuration. Here the separated flow zone extends to the trailing edge, but the intensity of the shock wave is large enough to dampen the oscillations.

Mode A2 of self-oscillations of two shock waves is also intermediate in terms of the Mach number of the incoming flow (fig.7).

The lower limit in terms of the Mach number at zero angle of attack is a stationary two-shock mode with curvilinear shock waves located symmetrically above the upper and lower surfaces of the profile. The points of separation and attachment are located on the streamlined surface.

The occurrence of self-oscillations of shock waves with an increase in the Mach number corresponds to the descent of attachment points from the profile surface and the formation of a combined separation zone. Here, instability of the combined separation zone arises, resembling Karman's vortex sheet behind a cylinder. Mode A2 of self-oscillations of shock waves is in a narrow range of Mach numbers of the incoming flow $\Delta M_\infty = 0.022$.

The upper limit in terms of the Mach number is the stationary regime with the same shocks configuration, but the intensity of the shock waves is sufficient to prevent the self-oscillations developing.

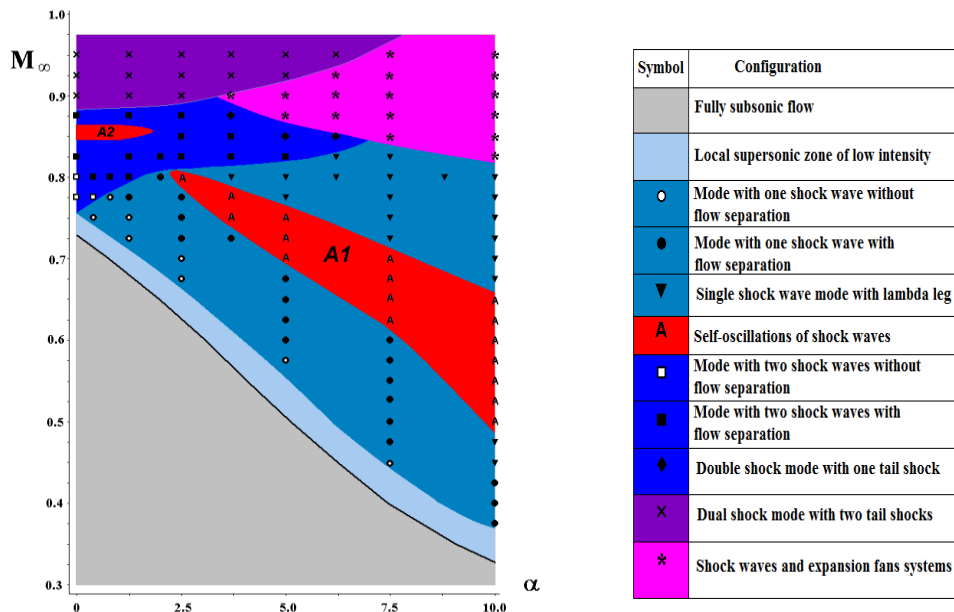


Figure 7: Refined "plan-map" of transonic modes turbulent flow around the NACA 0012 airfoil to that previously proposed in [6]

The refined “plan-map” of the turbulent flow regimes around the NACA 0012 airfoil presented in Fig.7 fully corresponds to the one obtained earlier in [6], but with the addition of the regime of two shock waves self-oscillations (A2 mode). Unfortunately, this mode was not found in the previous study, since it occupies a narrow range in Mach numbers and is located within the same zone of stationary shock wave configurations.

It should be noted that the self-oscillation modes zones A1 and A2 do not intersect. This is due to different physical reasons for their occurrence.

5 Conclusions

Based on the unsteady Reynolds-averaged Navier-Stokes equations, the possibility of the existence of a self-oscillation regime of two shock waves near the NACA 0012 airfoil at low angles of attack is shown. The physical features of the interaction of a shock wave with a turbulent boundary layer for the mode of self-oscillation of a shock wave on the NACA 0012 airfoil are analyzed. The influence of the Mach number of the incoming flow and the angle of attack on the evolution of aerodynamic coefficients is studied. A refined map of regimes of transonic turbulent flow around the NACA 0012 airfoil on the plane “Mach number of the incoming flow - angle of attack” is proposed.

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