

ADJOINT OPTIMIZATION OF AN AIRFOIL FOR LIGHT STOL FREIGHTER AIRCRAFT

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Abstract

An airfoil's lift coefficient (C_y) is a critical parameter determining the amount of payload an aircraft can carry. The lift-to-drag (L/D) ratio then determines the fuel efficiency and range with a defined payload. For those reasons, it is beneficial to optimize the airfoil used on the wings of an aircraft. In this paper, the adjoint optimization of the C_y of an airfoil for light freighter aircraft is discussed. The CASA C-212 Aviocar aircraft with its NACA 65(3)-218 airfoil was chosen for the optimization. The optimization uses the assumptions of inviscid incompressible potential flow, allowing the use of the panel method for flow solution. Verification computations imply, that the optimization results are relevant even in the compressible and viscous flow.

Keywords: Airfoil optimization, Adjoint method, Panel method

1 Introduction

The design optimization of an aircraft is a lengthy iterative process, as the aircraft design consists of many tasks and during each, the optimization can be applied. It is beneficial to employ optimization during the first stages of the design, as there are the least constraints, thus the optimization can yield the best results possible. However, at those stages, the computation time of the optimization should be kept as short as possible, because, during these stages, everything is still subject to change, and it might be needed to recompute the optimization many times. For those reasons, only the 2D airfoil optimization will be discussed in this article, as most of the 3D optimization problems are way too computationally demanding. Only in the last few decades, the advances in the computing power of available computers together with the evolution of optimization methods became significant enough to allow for objective function-based optimization in fluid mechanics.

In fluid mechanics, the adjoint optimization method took a prominent position because of its efficiency and accuracy [1] [2]. This optimization method, just like any other, needs a clearly defined set of design variables to operate upon. Those design variables were in this 2D airfoil optimization case provided by the parsec parametrization, as it is a proven method yielding good accuracy when describing general airfoil shapes [3] [4]. This article focuses on the adjoint optimization of an airfoil for light freighter aircraft in the aforementioned “early optimization” scenario with no additional constraints imposed. This article also provides conclusions about the optimization based on Xfoil data for the original and optimized airfoil.

2 Optimization method

To create a very quickly running procedure, that allows for the design optimization even in the first stages of the design, the panel method was used as a flow solver, combined with the adjoint method for the optimization. The panel method requires an inviscid incompressible potential flow for it to work. Still, it can be shown (see Section 2.2), that in the case of the optimization, it can also be used in the weakly compressible regime without any adjusting.

2.1 Panel method

The panel method is one way of solving potential flow problems. For the panel method, the inviscid incompressible potential flow must be assumed. Then it is possible to derive potential flow equations [5] and equations of the panel method [6]. It is possible to introduce dimensionless variables (1) and write down the equation for the cost function (2). It is also possible to derive the linear set of equations for the sources and vorticities on each panel from the impermeability of the airfoil's surface and the Kutta condition

[6]. This yields panel method equations presented in [6], which can be divided by the speed of undisturbed flow c_∞ to form panel method equations in their dimensionless form (4) and dimensionless tangent velocity on each panel (5). Those dimensionless equations are the starting point for the adjoint method. More details about potential flow and panel method, including explanation and derivation of governing equations can be found in [5] and [6]. In the equations below, c_t stands for the tangent velocity on the surface of the airfoil, p_∞ is the free-stream static pressure, ρ is the fluid density, L is the chord length (initially equal to one) and I is the cost function to be optimized. In the panel method, $\mathbf{w}$ is the vector of sources and vorticities on individual panels, Δx_i is the oriented length of i^{th} panel in the x direction, $\mathbf{A}$ is the control matrix of the panel method equations and $\mathbf{b}$ is the vector of the right-hand side. For further information about the definition and meaning of the symbols used (especially those related to the specific panel method implementation like θ , $\mathbf{A}$ etc.) we refer the reader to [6]. Letters in bold stand for vectors or matrices.

$$\hat{c}_t = \frac{c_t}{c_\infty}, \quad \hat{p}_\infty = \frac{2 \cdot p_\infty}{\rho \cdot c_\infty^2}, \quad \gamma = 1 + \hat{p}_\infty \quad (1)$$

$$L \cdot C_y = - \oint_l (\gamma - \hat{c}_t^2) \cdot dx \cong - \sum_{i=1}^N (\gamma - \hat{c}_{ti}^2) \cdot \Delta x_i = I \quad (2)$$

$$\Delta x_i = x_{i+1} - x_i, \quad x_{N+1} \equiv x_1 \quad (3)$$

$$\mathbf{A} \hat{\mathbf{w}} = \hat{\mathbf{b}}, \quad \hat{\mathbf{w}} = \frac{\mathbf{w}}{c_\infty}, \quad \hat{\mathbf{b}} = \frac{\mathbf{b}}{c_\infty}, \quad \mathbf{w} = [q_1, \dots, q_N, \Gamma]^T \quad (4)$$

$$\begin{aligned} \hat{c}_{ti} = \frac{c_{ti}}{c_\infty} = \cos(\theta_i - \alpha) + \sum_{j=1}^N \frac{\hat{w}_j}{2\pi} \left(\sin(\theta_i - \theta_j) \beta_{ij} - \cos(\theta_i - \theta_j) \ln \frac{r_{ij+1}}{r_{ij}} \right) + \\ + \frac{\hat{w}_{N+1}}{2\pi} \sum_{j=1}^N \left(\sin(\theta_i - \theta_j) \ln \frac{r_{ij+1}}{r_{ij}} + \cos(\theta_i - \theta_j) \beta_{ij} \right) \end{aligned} \quad (5)$$

2.2 Expansion into the weakly compressible flow

To use the panel method-based optimization for aviation purposes, it is necessary to expand it into the weakly compressible flow regime. This expansion stands on the assumption, that in the weakly compressible flow (considering $Ma_\infty \leq 0.5$ for this case), the Prandtl-Glauert (P-G) compressibility correction is valid [7]. This correction can be expressed in terms of equation (6), where C_p stands for a local pressure coefficient on the airfoil. Values with Tilda ($\sim$) are values in the compressible flow.

$$\widetilde{C}_p = \frac{C_p}{\sqrt{1 - Ma_\infty^2}}, \quad \widetilde{C}_y = \frac{C_y}{\sqrt{1 - Ma_\infty^2}} \quad (6)$$

The incompressible cost function can be rearranged to (7). Then it is possible to express the compressible lift coefficient (8), substitute the P-G correction (6) into it and get the compressible cost function (9), which is identically equal to the incompressible cost function (2). This means, that all equations used for the optimization in the incompressible regime also apply to the weakly compressible case, thus the appropriate region for this optimization can be expanded to $Ma_\infty \leq 0.5$.

$$I = L \cdot C_y = - \sum_{i=1}^N (\gamma - \hat{c}_{ti}^2) \cdot \Delta x_i = - \sum_{i=1}^N (1 - \hat{c}_{ti}^2) \cdot \Delta x_i = - \sum_{i=1}^N C_{pi} \cdot \Delta x_i \quad (7)$$

$$L \cdot \widetilde{C}_y = - \sum_{i=1}^N \widetilde{C}_{pi} \cdot \Delta x_i = - \sum_{i=1}^N \frac{C_{pi}}{\sqrt{1 - Ma_\infty^2}} \cdot \Delta x_i \quad (8)$$

$$\tilde{I} = L \cdot \widetilde{C}_y \cdot \sqrt{1 - Ma_\infty^2} = - \sum_{i=1}^N C_{pi} \cdot \Delta x_i = \sum_{i=1}^N (\hat{c}_{ti}^2 - \gamma) \cdot \Delta x_i = L \cdot C_y = I \quad (9)$$

2.3 Adjoint method with parsec parametrization

The adjoint method, while being very computationally efficient, has the disadvantage that it must be tailored for each use for a specific solver and specific cost function of interest. This is in contrast with a genetic algorithm or simplex method (and most other optimization methods), which are general algorithms that do not care about the expression for the cost function or about the solver used to obtain its value. This means,

that a lot of math has to be done when using the adjoint method as the optimization solver. In the end, it can be shown that it is necessary to solve the following equations (10) - (12). Mathematical background of the adjoint method can be found in [2], [8] and [9].

$$\mathbf{A}\hat{\mathbf{w}} = \hat{\mathbf{b}}, \quad \mathbf{A}^T \boldsymbol{\lambda} = -\left(\frac{\partial I}{\partial \hat{\mathbf{w}}}\right)^T \quad (10)$$

$$G_j = \frac{\partial I}{\partial F_j} + \lambda_l \cdot \frac{\partial R_l}{\partial F_j} \quad (11)$$

$$\mathbf{F}_{(n+1)} = \mathbf{F}_{(n)} + \zeta_0 \frac{\mathbf{G}}{\|\mathbf{G}\|} \quad (12)$$

The symbol $\mathbf{R}$ in (11) means the cancelled form of panel method equations (4), $\mathbf{G}$ stands for the gradient and $\boldsymbol{\lambda}$ stands for the vector of Lagrange multipliers. The use of unit gradient in (12) proved necessary for assuring the stability of the computation. The vector of parameters, $\mathbf{F}$, is defined by the parsec parametrization and is given in (13). The geometrical meaning of the parameters, together with the governing equations of parsec parametrization can be seen in [10] and [11].

$$\mathbf{F} = [r_{lo}, X_{lo}, Y_{lo}, Y_{xxlo}, r_{up}, X_{up}, Y_{up}, Y_{xxup}, \alpha_{te}, \beta_{te}, Y_{te}]^T \quad (13)$$

3 Original airfoil shape and acquisition of parsec parameters

3.1 Typical representative of STOL light freighter aircraft and its airfoil

As a typical representative of the STOL light freighter aircraft, the CASA C-212 Aviocar has been chosen. The aircraft has a max takeoff weight of 7700 kg, an empty weight of 3780 kg and 2950 kg of useful load capacity. In the passenger configuration, it can carry up to 28 passengers. All numbers mentioned above depend on the version of the aircraft. It has a boxy fuselage, fixed landing gear, a high-mounted wing and a conventional tail. It is designed to operate in stark conditions and from unpaved airfields. It serves both as a civilian and military aircraft. In the civilian sector, it is mainly used as a short-haul cargo or passenger aircraft. For its wings, this aircraft uses the NACA 65(3)-218 airfoil from root to tip [12]. In publicly available sources, cruise speeds vary from 83 m/s [13] to 107 m/s [14]. The most important characteristics of this aircraft in cruise can be seen in Table 1. The cruise speed was estimated as 100 m/s, and the cruise altitude was chosen as 3 km because the aircraft lacks a pressurization system. The temperature in the cruise altitude was estimated as -3 °C, and the air viscosity at this temperature was then calculated following equations in [15].

Table 1: Characteristics of CASA C-212 Aviocar [13] [14] [15] [16]

Chord length [m]	2
Cruise speed [$m \cdot s^{-1}$]	100
Cruise altitude [km]	3
Cruise altitude temperature [°C]	-3
Speed of sound at cruise altitude [$m \cdot s^{-1}$]	330
Kinematic viscosity of air at cruise altitude [$m^2 \cdot s^{-1}$]	$1.3 \cdot 10^{-5}$
Ma in cruise [1]	0.3
Re in cruise [1]	$15.38 \cdot 10^6$

3.2 Initial parameters of the airfoil

The initial parsec parameters were identified using the in-house MATLAB fmincon function. This function is a local, gradient-based optimization algorithm, that chooses between 4 optimization methods [17]. Further details about the fmincon function are available in [17]. The measure of optimality was chosen as the sum of squared errors at the definition points obtained from the airfoil source file in Selig format. The coordinates of definition points are available in [18]. The identified parsec parameters are presented in Table 2 Col. 2. The comparison of the identified and design shape can be seen in Figure 1 (axes not to scale). From this figure, it can be seen, that the approximation by parsec parameters presented in Table 2 is of a very good quality.

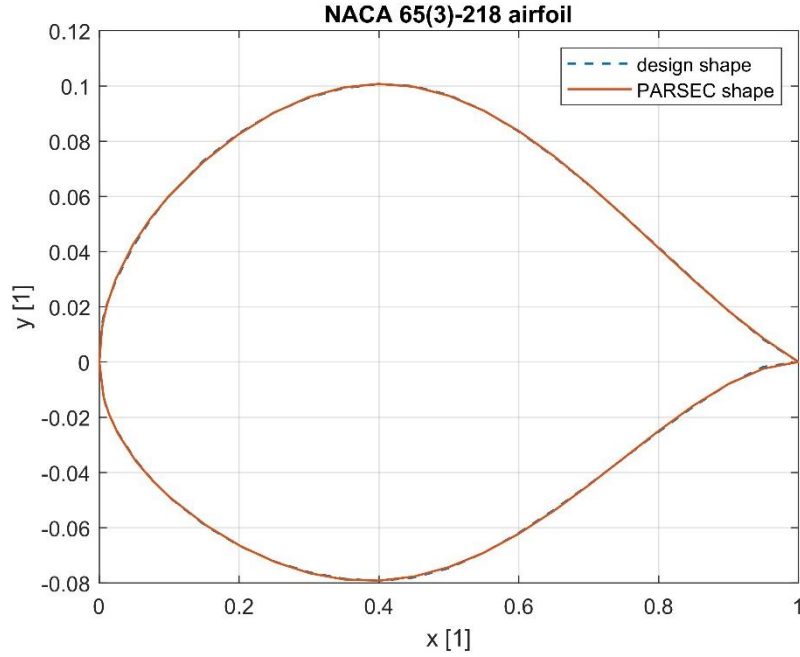


Figure 1: Agreement of the design shape and parsec-approximated shape of the NACA 65(3)-218 airfoil

4 Results

4.1 Optimization settings and comparison of original and optimized geometry

The optimization was run from the starting point defined in Table 2 Col. 2 at the fixed nominal angle of attack (AoA) of 2 deg for a maximum of 50 iterations with iteration step $\zeta_0 = 0.0002$. The convergence criteria were the following: (a) Absolute improvement to C_y of 0.2 and (b) Relative improvement to C_y of 25 %. The optimization halts on reaching the first of those criteria. The optimization reached the (b) criterion in the 43rd iteration. On top of that, there was a stopping criterion for reaching the defined shape change. The shape change is measured as the root mean square (RMS) difference from the original shape and is calculated for the nodes of the panels. The allowed value of RMS shape change was set to $\Delta Y_{RMS} = 0.005$ for this optimization. Identified and optimized parameters can be seen in Table 2 below, together with the lift coefficient of the original NACA 65(3)-218 airfoil and its optimized variant. The RMS shape change and absolute and relative changes to C_y are also presented.

Table 2: Parsec parameters, lift coefficient and shape difference

	Identified	Optimized
α [°]	2	2
r_{lo} [1]	0.01299	0.01360
X_{lo} [1]	0.3863	0.3861
Y_{lo} [1]	-0.07927	-0.07579
Y_{xxlo} [1]	0.7701	0.7702
r_{up} [1]	0.01969	0.01959
X_{up} [1]	0.4026	0.4028
Y_{up} [1]	0.1008	0.1042
Y_{xxup} [1]	-0.9129	-0.9128
α_{te} [°]	-3.994	-4.043
β_{te} [°]	9.184	9.182
Y_{te} [1]	4.317e-05	-0.006708
ΔY_{RMS} [1]	---	0.0039
C_y [1]	0.4441	0.5574
ΔC_y^{ABS} [1]	---	0.1133
ΔC_y^{REL} [%]	---	25.51

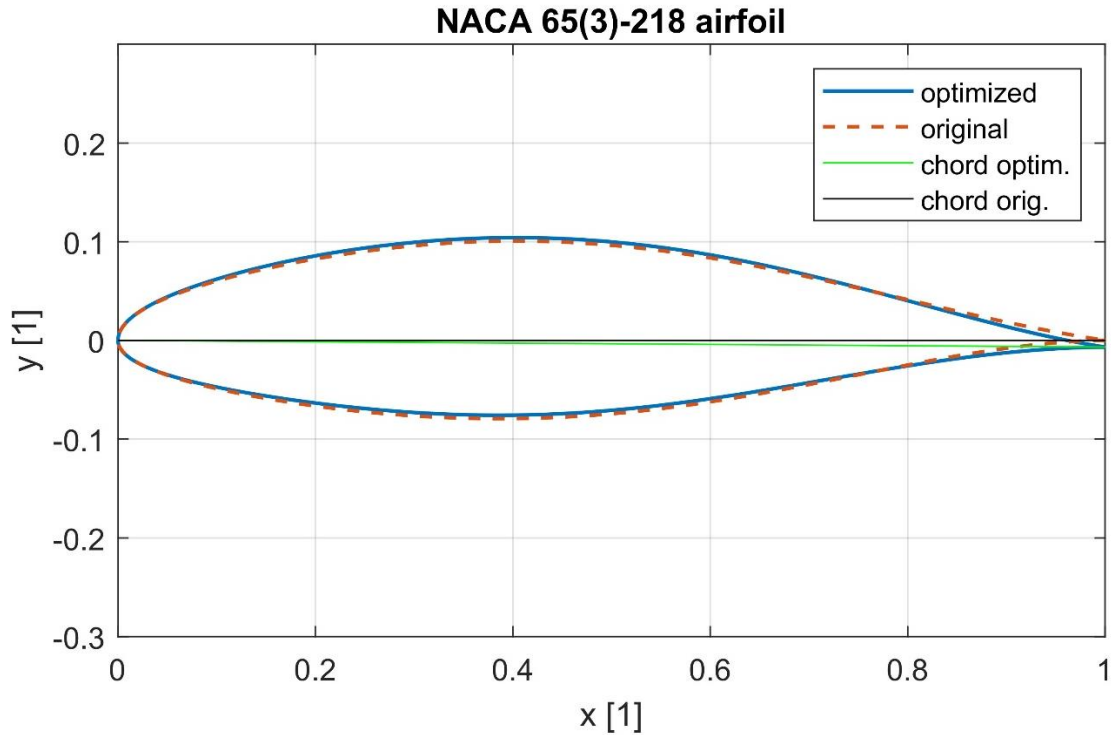


Figure 2: Original and optimized shapes, the optimized shape has 25.5 % more lift than the original

4.2 Performance at cruise speed and altitude

After the optimization, verification computation in Xfoil was carried out. The Re and Ma numbers were set according to the obtained cruise speed and altitude data in Table 1 and AoA was set to 2 deg. The main objective of the verification was to judge, whether the convergence criteria (absolute improvement to C_y of 0.2 or relative improvement of 25 %) have been met in the compressible and viscous scenario as well. From Table 3 it can be stated that the convergence criteria have been met and that the benefit of the optimization (in terms of C_y) is even bigger than in the incompressible inviscid scenario.

Table 3: Xfoil results for the adjoint-optimized NACA 65(3)-218 airfoil, $Re = 15\,380\,000$, $Ma = 0.3$

Type of Xfoil simulation	Incompressible and inviscid				Compressible and viscous			
	Original airfoil	Optimized airfoil	Absolute difference	Relative difference	Original airfoil	Optimized airfoil	Absolute difference	Relative difference
α [°]	2	2	---	---	2	2	---	---
C_y [1]	0.4471	0.5626	0.1155	+25.83 %	0.4294	0.5497	0.1203	+28.02 %
C_x [1]	---	---	---	---	0.0045	0.00475	0.00025	+5.56 %
L/D [1]	---	---	---	---	95.5	115.63	20.13	+21.08 %
C_M [1]	- 0.0511	- 0.0678	- 0.0167	- 32.68 %	- 0.0448	- 0.0623	- 0.0175	- 39.06 %

5 Conclusion

In this paper, the optimization of an airfoil used on STOL light freighter aircraft was discussed. A typical representative of this category in the form of CASA C-212 Aviocar was selected. Then the characteristics of this aircraft necessary for the optimization were looked up. Regarding the optimization algorithm, it can be concluded that all parts worked as designed and expected. The parsec parameters of NACA 65(3)-218 airfoil were successfully identified from a file containing airfoil shape in Selig format. This was done by using the `fmincon` function available in MATLAB. Then those parameters were passed to the adjoint optimizer, which is a code written by the authors in MATLAB. The adjoint optimization performed as expected, reaching convergence criteria in the 43rd iteration, while making minimal changes to the airfoil shape. The Xfoil verification results confirmed the benefit of the optimization and showed, that the L/D ratio was significantly improved as well because the changes did not affect the drag coefficient much.

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