

SYNCHRONIZATION OF AIRFLOWS AT VARYING HEIGHTS

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Abstract

We propose a method to study the dynamics of energetic events in airflows at different heights. The method is based on a mathematical model for invariant pattern recognition by a visual system, which is combined with a wavelet analysis method. Here we use the first four Krawtchouk functions as the mother wavelets. The method allows to determine the time-frequency characteristics of the local impulse components for continuous signals. These extracted impulse components of airflow velocities can be effectively utilized in cross-correlation analysis aimed at identifying potential synchronizations among airflows at different heights.

Keywords: Krawtchouk functions, wavelet, turbulent flow, uncertainty principle.

1 Introduction

In [1] it was shown that coherent structures play a dominant role in momentum and mass transport in turbulent boundary layers. Wind tunnel measurements have revealed large organized structures in the form of regions of low and high momentum or clusters of correlated longitudinal and vertical velocity fluctuations (e.g. sweep and ejection events). These structures are not eddies; they have the form of irregular waves [3].

One of the widespread methods for studying non-stationary processes is the windowed Fourier transform. It allows to explore the frequency characteristics of a signal in a certain time interval. The signal is varied to extract a relatively small fragment of it at a specific time to study its frequency properties. However, if a very small fragment is extracted, its bandwidth will be artificially large to achieve good temporal localization due to the uncertainty principle. Consequently, the spectral properties will have little to do with the spectral properties of the signal [4].

The uncertainty principle is a fundamental result of signal analysis. It sets a lower bound on the simultaneous localization of a signal in both the time and frequency domains. This basic principle illustrates the impossibility of circumventing nature by precisely measuring the frequency of an input signal with arbitrary accuracy within a finite time interval [5], [6], [7].

Following [8] let us consider the uncertainty principles in more detail. This principles state that a function and its Fourier transform cannot be sharply localized at the same time. To be more precise, if we consider a dimension $d \geq 1$ and denote the scalar product as $\langle \cdot, \cdot \rangle$ and the Euclidean norm on $\mathbb{R}^d$ as $\|\cdot\|$, for $f \in L^2(\mathbb{R}^d)$, we define the Fourier transform of f as follows:

$$\hat{f} = \int_{\mathbb{R}^d} f(t) e^{-2i\pi \langle t, y \rangle} dt. \quad (1)$$

The most famous uncertainty principle created by Heisenberg and Weil can be presented in the following inequality named after Heisenberg: For any $f \in L^2(\mathbb{R})$ and any $a, b \in \mathbb{R}$

$$\inf_{a \in \mathbb{R}} \left(\int_{\mathbb{R}^d} (x_i - a)^2 |f(x)|^2 dx \right) \inf_{b \in \mathbb{R}} \left(\int_{\mathbb{R}^d} (\xi_i - b)^2 |\hat{f}(\xi)|^2 d\xi \right) \geq \frac{\|f\|_{L^2}^4}{16\pi^2}. \quad (2)$$

Following [8], [10] equality is possible if and only if f has the form

$$f(x) = C e^{2i\pi b x} e^{-\alpha(x-a)^2} \quad (3)$$

for some constants $C \in \mathbb{C}$, $\alpha > 0$ and $a, b \in \mathbb{R}$ (see (2)).

Time-frequency representations play a crucial role in effectively describing signals that vary over time, encompassing those with time-varying frequencies [5]. The method of wavelet transforms which allows the decomposition of functions using a fixed orthogonal family of functions of constant form, different scales and locations, has recently gained great importance in the analysis of signals and experimental data related to various physical phenomena [13],[5]. The continuous wavelet transform (CWT), a well-established tool employing wavelets as its basis, introduces a scale variable alongside the time variable. This transformation generates templates through translations and dilation of the basic wavelet. Offering variable time localization and frequency resolution, the CWT stands out as one of the premier methods for analyzing unsteady signal analysis [14]. In turbulent flow analysis, wavelet analysis proves instrumental in identifying high-energy and repeating low-frequency structures, particularly by utilizing the Morlet function as the mother wavelet, and in detection of eddies by utilizing the Mexican hat function [2].

The Heisenberg uncertainty principle imposes a constraint on the ability to simultaneously adjust temporal and spectral resolution [14]. This principle dictates the time-frequency resolution, leading to a trade-off between time localization and frequency resolution, preventing both domains from achieving their maximum levels simultaneously [9]. When the mother wavelet is characterized by short time span and high frequency, simultaneous information about time and frequency can be precisely determined. However, as the wavelet undergoes stretching, its time span extends while its frequency decreases. In [11] an overview was presented regarding the Heisenberg uncertainty principle and its correlation to the decomposition level of Discrete Wavelet Transformation.

Gabor atoms are constructed by translating a time window g in time and frequency [13]:

$$g_{u,\xi}(t) = g(t - u)e^{i\xi t}. \quad (4)$$

The energy of $g_{u,\xi}$ tends to be concentrated in the neighborhood of u within an interval of size σ_t measured by the standard deviation of $|g|^2$. Its Fourier transform represents a translation by ξ of the Fourier transform $\hat{g}$ of g :

$$g_{u,\xi}(\omega) = g(\omega - \xi)e^{-iu(\omega - \xi)}. \quad (5)$$

The energy of $g_{u,\xi}$ is therefore localized near the frequency ξ within an interval of size σ_ω , which accounts for the domain where $g(\omega)$ is non-negligible. When depicted in a time-frequency plane (t, ω) , the energy distribution of the atom $g_{u,\xi}$ is symbolically depicted by the Heisenberg rectangle, as shown in Figure 1,b. This rectangle, centered at (u, ξ) , exhibits a temporal width σ_t and a frequency span of σ_ω . According to the uncertainty principle, its area satisfies

$$\sigma_t \cdot \sigma_\omega \geq \frac{1}{2}. \quad (6)$$

This area is minimal when g takes the form of a Gaussian function, leading to the atoms $g_{u,\xi}$ being referred to as Gabor functions [13].

This Heisenberg Principle is in the very nature of things and has nothing to do with the numerical accuracy of wavelet analysis [11]. Figure 1,a depicts the energy distribution of two windowed Fourier Transformation [13]. Consequently, the Heisenberg principle posits the impossibility of achieving simultaneous temporal and spectral resolution in both Fourier and wavelet analysis.

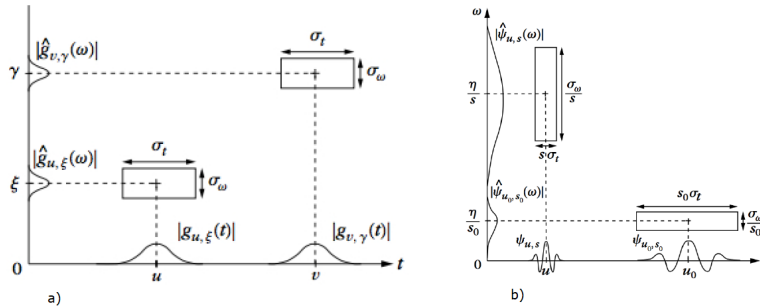


Figure 1: Energy distribution of a) two windowed Fourier Transformation and b) two Wavelets [13], [11].

2 The method to analyze the velocity at varying heights

In [15] we introduced a wavelet analysis method for localization, intended for analyzing the dynamics of signals of diverse natures. The method aims to find the time-frequency characteristics of local impulse components within continuous signals. Consequently, we separate the information about the impulse shape from its time of occurrence. The method combines a mathematical model for invariant pattern recognition by a visual system with a wavelet analysis approach, employing the first four Krawtchouk functions as the mother wavelets (Figure 2).

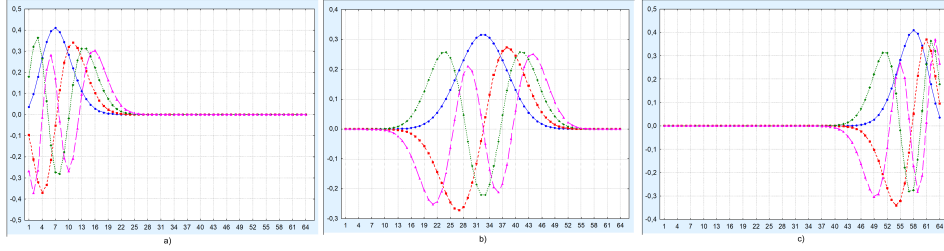


Figure 2: First four Krawtchouk functions for values a) $p = 0.1$, b) $p = 0.5$, c) $p = 0.9$; $i = \overline{1, 64}$.

According to this method the analysis of the velocity of the airflows consists of the next steps:

Step 1. We define the scale of the wavelet transform using windows of length N points, where $N = 2^n$ and n ranges from 10 to 2. We initiate this process with window width of 1024 points.

Step 2. We generate a new zero level for the input signal $y(i), i = \overline{1, 1024}$ within the largest window: $y(i, a_0) = y_{\text{input}}(i, a_0) - \bar{y}_{\text{input}}(i, a_0)$, for $i = \overline{1, 1024}$. Here, $\bar{y}_{\text{input}}(i, a_0) = \sum_{i=1}^{1024} y_{\text{input}}(i, a_0) / 1024$ is the average illumination for the largest window (Figure 3).

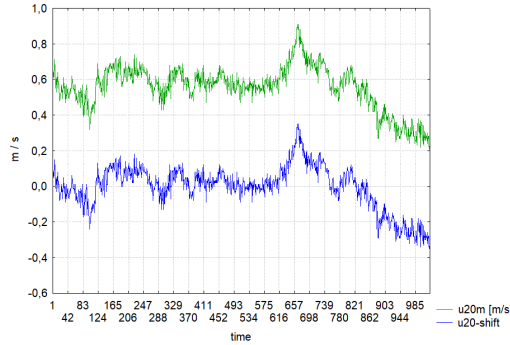


Figure 3: Input signal $y(i), i = \overline{1, 1024}$ (green line) and shifted signal with its new zero level (blue line).

Step 3. We compute the first four Krawtchouk functions for windows of sizes $N = 2^n$, where n ranges from 10 to 2 and p takes values of 0.1, 0.2 up to 0.9. This choice encompasses nine asymmetry coefficients of the Krawtchouk functions, allowing for a more accurate approximation of non-symmetrical impulse components.

Step 4. We compute the generalized spectral coefficients relative to the set of shifts within the basis of Krawtchouk functions:

$$c_k^{(p)}(a, a_0) = y(i, a_0) * F_k^{(p)}((i - a) \bmod N), \text{ where } i, a = \overline{1, N}; k = 1, 2, 3, 4; p = 0.1, 0.2, \dots, 0.9.$$

Step 5. We compute the energy functional

$$W^{(p)}(a, a_0) = \sum_{k=1,2,3,4} |c_k^{(p)}(a, a_0)|^2.$$

Step 6. We find the maximum of the energy functional

$$\max_{p=0.1, \dots, 0.9; a=\overline{1, N}} W^{(p)}(a, a_0) = W_{\max}^{(p_0)}(a_0, a_0).$$

Here, we find the temporal localization of the signal's impulse component and the asymmetry coefficients of the Krawtchouk basis functions, which maximally correspond to the asymmetry of the impulse.

Step 7. Now, we ascertain whether the extremum identified in the prior step should be interpreted as a "reaction" or as background fluctuations. If $W_{max}^{(p_0)}(a_0, a_0) \geq T_n$, where T_n represents the threshold value of the energy functional for the window at level n , we define this part of the signal as reaction and proceed to the next step. These threshold values T_n are predetermined using a training sequence provided by signal analysis experts.

We analyzed the velocity of the airflows at 20 meters, 40 meters, 60 meters, and 80 meters. Subsequently, we determined the threshold values of the energy functionals $T[n]$ for the windows with lengths of powers of 2^n , where $n = 10, 9, \dots, 2$:

height of airflow	$T[10]$	$T[9]$	$T[8]$	$T[7]$	$T[6]$	$T[5]$	$T[4]$	$T[3]$	$T[2]$
20 m	0.589	0.249	0.119	0.089	0.069	0.039	0.029	0.90	0.90
40	2.019	0.639	0.269	0.099	0.079	0.049	0.029	0.90	0.90
60m	3.209	1.409	0.639	0.229	0.119	0.059	0.039	0.90	0.90
80m	3.199	1.129	0.449	0.199	0.129	0.069	0.059	0.90	0.90

Step 8. We identify the time of reaction (impulse) within window number l at level n : $a_l^{n_1} = a_0, n_1 = n, 2; l = \overline{1, S_n}$ and determine the amplitudes and frequencies of reaction (impulse) within window number l at level n : $c_k^{(p_0)}(a_0, a_0)_{k=1}^4, n_1 = \overline{n, 2}; l = \overline{1, S_n}$.

Step 9. We restore the impulse component of the signal

$$\tilde{y}(i) = \sum_{k=1,2,3,4} c_k^{(p_0)}(a_0, a_0) F_k^{(p_0)}((i - a_0) \bmod N), i = \overline{1, N}.$$

Step 10. We perform the subtraction of the reconstructed signal from the input signal: $y_l^n = y(i, a_0) - \tilde{y}(i); i = \overline{1, 2^n}$. Consequently, we consider low-frequencies oscillations as the drift of the isoline for high-frequencies oscillations. Moreover, this process eliminates the redundant identification of frequency characteristics pertaining to the same impulse component in subsequent windows of the wavelet analysis. This particular approach stands out as a distinguishing features of our algorithm compared to other wavelet transform algorithms.

Step 11. We proceed to the subsequent window $l = l + 1$ at this level n of the wavelet analysis. If the remainder of the signal is smaller than the window width, we pad the signal with zeros: if $M < (S_n - 1) \cdot 2^n$, then $y(i) = 0, i = \overline{M, S_n \cdot 2^n}$. When we have reached the end of the sequence of windows for a given level $l > S_n$, we transition to the next level of windows $n = n - 1$. If $n < 2$, the algorithm reaches the end.

As a result, we obtain the decomposition of the input signal y into the impulse component y_{impuls} and the residual y_{residual} . In Figure 4 we display the velocity of the airflows at 20 meters (blue line), its impulse component (red line) and the residual - the noise (green line).

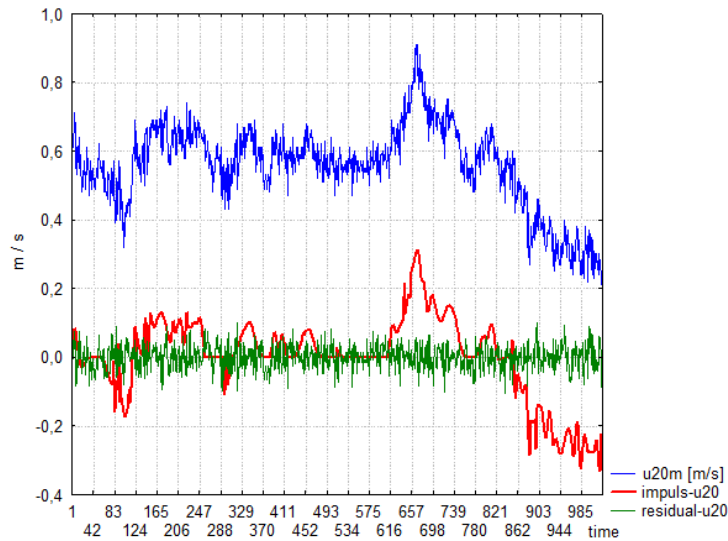


Figure 4: Input signal: the velocity of the airflows at 20 meters (blue line), its impulse component (red line) and the noise (green line).

In Figures 6 - 7 we present the results of our comparative analysis of the spectrograms detailing airflow velocities at heights of 20, 40, 60 and 80 meters. The spectrograms were calculated using wavelet analysis for localization b), e) and classical wavelet analysis c), f) employing the first four Krawtchouk functions as the mother wavelet in both methods. The red lines depict the impulse components of the respective velocities, providing a good approximation of the input signals. The residuals, representing the removed noise, are presented in Figure 5. The differences between clas-

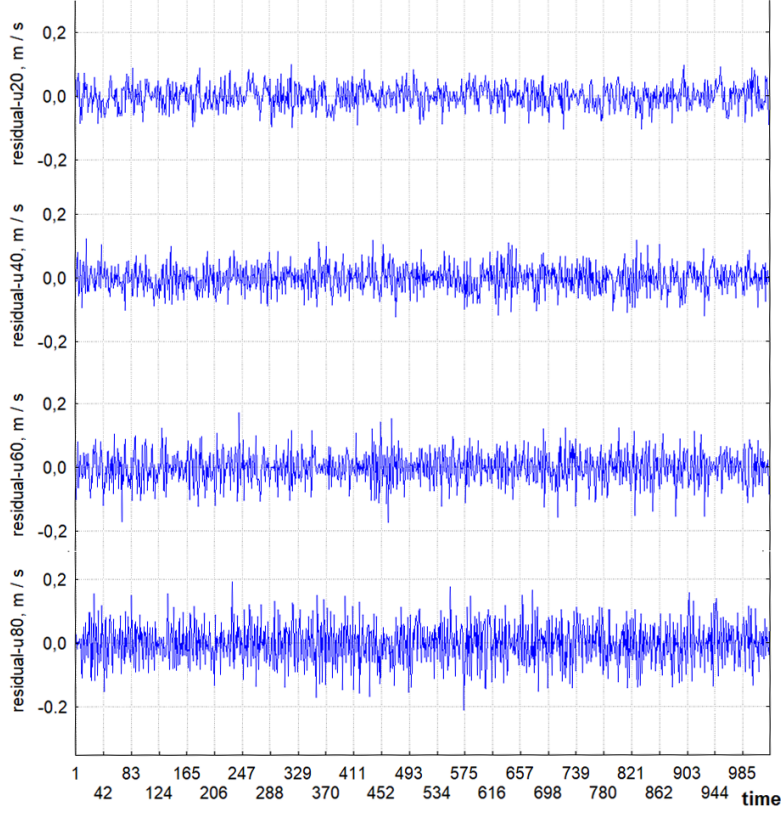


Figure 5: Residuals (noise) of the velocities of the airflows at 20, 40, 60 and 80 meters.

sical wavelet analysis and our proposed wavelet analysis for impulse component localization are as follows. In classical wavelet analysis, the wavelet spectrogram of a signal is built by the scalar product of the signal with the daughter wavelets within the corresponding scale windows, determining the amplitude of the frequency components. Figures 6-7 *c – f* show the spectral powers of the signals, represented as the sums of the squares of the amplitudes resulting from the signal decomposition using the first four Krawtchouk functions within the scale windows w_k ($k = 10, 9, \dots, 2$) with a length of 2^n points ($n = 10, 9, \dots, 2$). In the context of wavelet analysis for the localization of the impulse component, we build wavelet spectrograms based on the energy functional. This functional is defined as the sum of the squares of the convolution of the signal with the first four Krawtchouk functions, each shifted by the parameter t ($t = 0, 1, \dots, 2^n$, $n = 10, 9, \dots, 2$). As a result, the largest energy peak representing the maximum of the functional, coincides with the localization of the maximum signal peak. Figures 6-7 *b – e* show the spectral powers of the signals in terms of the energy functional within the scale windows $wink$ ($k = 10, 9, \dots, 2$) with a length of 2^n points ($n = 10, 9, \dots, 2$) [15]. The calculated spectrograms of airflow velocities at heights of 20, 40, 60 and 80 meters using wavelet analysis for localization give high accuracy in resolving energy events within the frequency domain and demonstrate precise time localization. In comparison, the spectrum derived with classical wavelet analysis (Figures 6 - 7 *c, f*) appears fuzzy, particularly within the low-frequency bands.

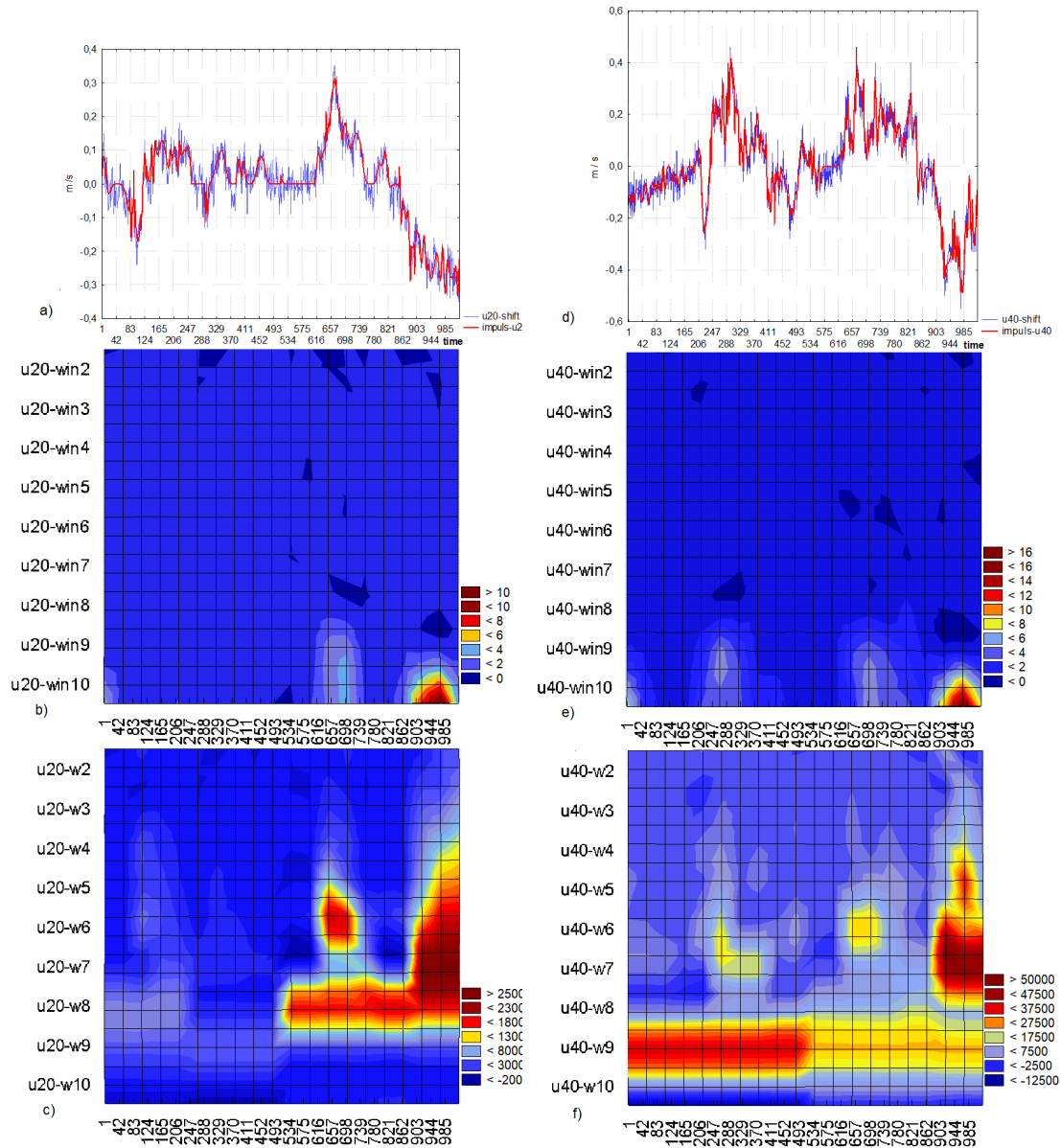


Figure 6: Input signal: the velocity (blue line) and impulse component (red line) of the airflows at a) 20 meters and d) 40 meters; wavelet spectrogram calculated using the wavelet analysis method for localization for the velocity at b) 20 meters and e) 40 meters compared to the classical wavelet spectrogram at c) 20 meters and f) 40 meters.

The identified impulse components of the airflow velocities at varying heights were employed in the cross-correlation analysis. Cross-correlation is commonly utilized for assessing the relationship between two distinct time series. It measures the similarity, expressed as correlation, between the first time series x and altered (shifted or lagged) replicas of the second time series y as a function of the lag. The goal is to identify the leading and lagging variable. We can consider the x -variable as the leading variable of the y -variable when utilizing x -variable values to predict future values of y . To assess this relationship, we computed the correlation (corr) with the standard error (Std.Err.) between the two time series x and $y \pm k$, where $\pm k$ represents a lag. For the velocities at 80 and 60 meters, the results are as follows: lag = 19, corr = 0.55, Std.Err. = 0.032; for the velocities at 60 and 40 meters: lag = 229, corr = 0.36, Std.Err. = 0.035; for the velocities at 40 and 20 meters: lag = 9, corr = 0.63, Std.Err. = 0.031.

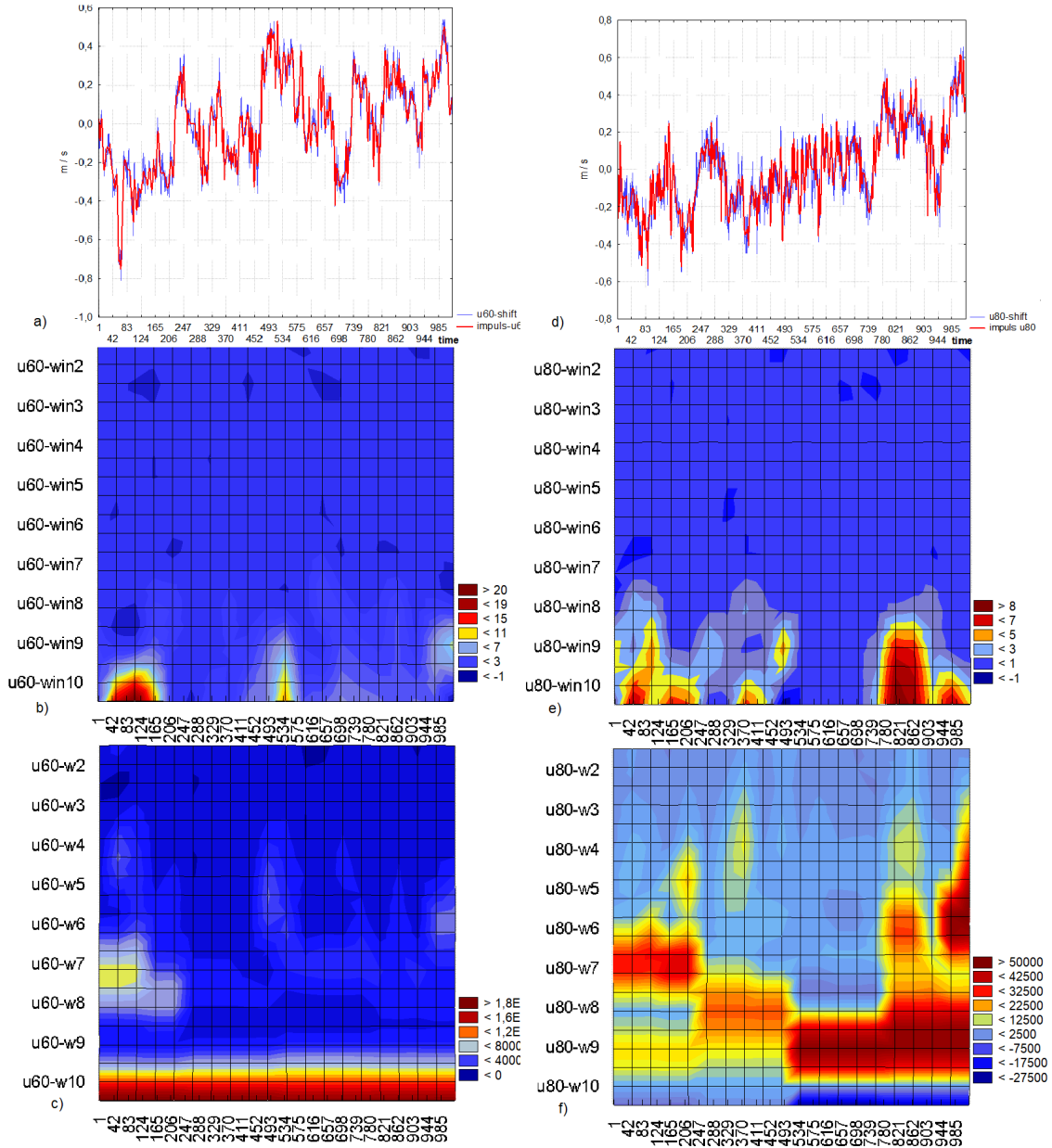


Figure 7: Input signal: the velocity (blue line) and impulse component (red line) of the airflows at a) 60 meters and d) 80 meters; wavelet spectrogram calculated using the wavelet analysis method for localization for the velocity at b) 60 meters and e) 80 meters compared to the classical wavelet spectrogram at c) 60 meters and f) 80 meters.

3 Conclusions

We proposed a method to study the dynamics of energetic events within airflows at varying heights. The extracted impulse components of airflow velocities can be used in cross-correlation analysis to explore synchronization among flows at different heights.

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