

QUASI-STATIONARY MATHEMATICAL MODEL OF HYBRID ROCKET ENGINE COMBUSTION CHAMBER

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Abstract

This paper presents a quasi-stationary one-dimensional mathematical model of the combustion chamber of a hybrid rocket engine. The model is based on stationary balance equations, considering time-varying input parameters. It makes it possible to quickly estimate the variability of parameters such as pressure, velocity, or temperature of gases inside the combustion chamber during engine operation. In addition, the assumption of a global paraffin oxidation reaction in nitrous oxide allows the determination of the composition of the exhaust gases. The results are consistent with numerical data from other researchers available in the literature.

Keywords: hybrid rocket, mathematical modeling, oxidation, paraffin, nitrous oxide.

1 Introduction

Conventional rocket fuels are liquid or solid fuels. Today, a growing body of work in the aviation literature is concerned with experimental and theoretical research into the possibility of improving the efficiency and reducing the cost of rocket flight by using hybrid systems in which the fuel and oxidizer are in two different phases (solid and liquid) [1–5]. The main criteria for the use of this type of rocket propulsion are the simplicity of the system itself and the associated lower cost and operational safety of the entire device. The advantages of hybrid engines over solid-fuel engines are easier control of operation, easier transportation and safer storage of non-explosive fuel. The disadvantage, on the other hand, is lower thrust due to poorer mixing of combustion products.

The purpose of this paper is to present a method for numerical modeling of the combustion chamber of a hybrid rocket engine as a simple tool for the initial design stage to improve the efficiency and quality of the fuel combustion process in an aircraft rocket engine during flight.

2 Mathematical model

The general assumption of the model is that for each time step of the simulated flight process, the phenomena occurring in the combustion chamber are treated stationary. Due to the simplicity of the rocket engine design (a tube filled both on the outside and in the core with solid fuel, see Fig.1), the mathematical model can be simplified to the one-dimensional case, where the privileged direction is the direction of gas flow along the rocket. Such approach can give the solution as fast as the simplest 0D models [3,6] but with more information about the changing of the parameters along the rocket length, and it simultaneously is not such time demanding as 2D or 3D ones using full turbulence and chemical kinetics modeling.

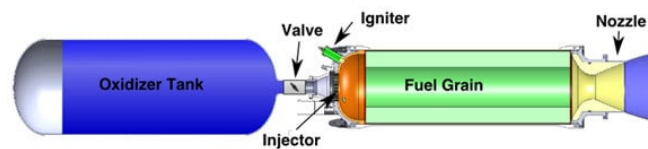


Figure 1: Scheme of the hybrid rocket engine, according to [7]

The model is therefore based on one-dimensional equations of mass, momentum and energy balance of gases, respectively [8]:

$$\frac{1}{A} \frac{d}{dz} (\rho w A) = \dot{\Omega} \quad (1)$$

$$\frac{1}{A} \frac{d}{dz} (\rho w w A) + \frac{dp}{dz} = -\tau_w \frac{C}{A} + \rho g \quad (2)$$

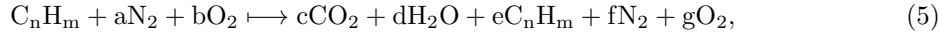
$$\frac{1}{A} \frac{d}{dz} (\rho w u A) = -\frac{p}{A} \frac{d(wA)}{dz} + \dot{q}_{rad} \frac{C}{A} + \tau_w \frac{C}{A} w + \dot{Q}_{comb}^v \quad (3)$$

where ρ is the density of the gases [m³/kg], w their velocity [m/s], p the pressure [Pa], u the specific energy [J/kg], z the position along the flow [m], A the cross-section area of the chamber [m²], C the wetted perimeter [m], g the gravitational acceleration [m²/s], $\dot{Q}_{comb}^v$ is the volumetric heat flux delivered to the system due to combustion [W/m³]. The pressure loss due to friction τ_w [Pa] can be defined as:

$$\tau_w = f \frac{\rho w^2}{2}, \quad (4)$$

where f is a friction factor.

The solid-gas phase transition is modeled by the source term $\dot{\Omega} = f(\dot{m}_{ox})$ [kg/(m³s)], defined by the so-called regression function of solid fuel mass loss due to reaction with the oxidant [9,10]. Based on previous work [11,12], it can be assumed that after leaving the tank, the two-phase (liquid-vapor) oxidant mixture in the form of nitrous oxide expands and finally enters the combustion chamber in gaseous form. Therefore, the oxidation reaction of paraffin (gas) in an atmosphere of N₂O (gas) can be written as:



whereby, for simplicity, it is assumed that it occurs instantaneously, i.e. the kinetics of the chemical reaction are not included in the calculations. The heat delivered to the chamber due to oxidation is determined from the regression function of fuel loss as

$$\dot{Q}_{comb} = \dot{m}_{fuel} h_{comb}, \quad (6)$$

whereby the enthalpy of combustion of solid fuel here is $h_{comb} = 42$ MJ/kg. Analyzing the system chemically, the mass balances of the individual components are additionally considered:

$$\frac{1}{A} \frac{d}{dz} (\rho w A Y_k) = \dot{\Omega}_k, \quad \dot{\Omega} = \sum_{k=1}^{ns} \dot{\Omega}_k, \quad (7)$$

where Y_k is the mass fraction of the k -th component in the mixture, ns is the number of considered chemical components. The source terms for each component $\dot{\Omega}_k$ are determined from the global oxidation reaction of paraffin (C₃₀H₆₂). Other heat sources present in the energy balance equation (3) remain to be determined. The heat absorbed from outside is neglected, while the heat lost due to radiation ($\dot{q}_{rad}$, [W/m²]) is determined using the Stefan-Boltzman formula [13]:

$$\dot{q}_{rad} = -\sigma \epsilon (T^4 - T_{wall}^4), \quad (8)$$

where σ is the Stefan-Boltzman constant [W/(m²K⁴)], ϵ is the emissivity.

To close system of equations the gas mixture is treated as a perfect gas described by the equation of state:

$$\frac{p}{\rho} = \frac{R_i}{M_{mix}} T, \quad (9)$$

where M_{mix} is the mixture molar mass [kg/kmol] defined as:

$$M_{mix} = \frac{1}{\sum_k Y_k / M_k}, \quad (10)$$

and M_k is the molar mass of k -th substance. It is easy to rewrite the energy balance equation (3) in the formula with temperature T instead of specific internal energy u by using the definition of specific enthalpy h [J/kg]:

$$h = u + pv = u + p/\rho, \quad (11)$$

which for gas can be also calculated from:

$$h = c_p T, \quad (12)$$

where c_p [J/(kgK)] is the mass specific heat at constant pressure. For gas mixture this can be defined as:

$$c_p = \sum_k Y_k c_{pk}, \quad (13)$$

where an individual heat capacities can be estimated basing on NASA polynomials [14]. By differentiating the equation of state (9) the following expression:

$$\frac{d\rho}{dz} = \left(\frac{\partial \rho}{\partial p} \right) \bigg|_T \frac{dp}{dz} + \left(\frac{\partial \rho}{\partial T} \right) \bigg|_p \frac{dT}{dz} \quad (14)$$

can be used in the set of basic balance equations (1-3) with a vector of unknowns:

$$\mathbf{X}^T = [p, T, w] \quad (15)$$

reducing it to a system of linear equations with its derivatives as new unknowns:

$$\mathbf{A} \mathbf{Y}^* = \mathbf{B}, \quad \mathbf{Y}^* = \frac{d\mathbf{X}}{dz}, \quad (16)$$

whereby heat lost due to radiation is treated explicitly. What remains to be calculated is the vector of unknowns $\mathbf{X}$, which can be determined using the Runge-Kutta method for solving a system of ordinary differential equations [15].

For given time-varying input profiles of velocity, pressure and temperature, the distribution of these parameters along the combustion chamber are calculated. In addition, the distribution and change over time of the chemical compounds concentration is also obtained. The corresponding procedures were prepared in the FORTRAN90 language and implemented into an in-house numerical code.

3 Results

For the given varying profiles of pressure and oxidant mass outflow from the nitrous oxide tank [11, 12] (Fig. 2), the distribution of gas mixture parameters along the combustion chamber length were obtained.

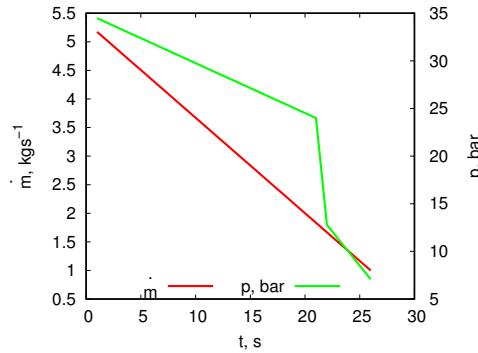


Figure 2: Time varying initial boundary conditions for mass flow rate of nitrous oxide and its pressure.

The combustion chamber with diameter of 0.1m and length of 0.7m were considered. Additionally, two paraffin solid fuel sources placed in the core and at chamber internal walls were taken into account. Fig. 3 shows the variation of selected parameters in time and space (along the axis of the rocket engine). As can be seen, the maximum temperature of exhaust gases exceeds 2700°C and occurs after 3s of the flight (Fig. 3 left graph), and at this time the maximum gas velocity takes place and its value is about 160m/s (Fig. 3 right graph). Therefore, based on the presented

assumptions, it can be concluded that this time corresponds to the most intense combustion process of the rocket fuel. This also manifests itself in the distribution of water vapor content in the exhaust gas - its maximum value is about 12.5% by weight for 3s of the process (Fig. 4 right graph). After that time, the values of the mentioned parameters decrease, which is associated with the burning of the fuel source (core and external fuel grain). There is, however, an increase in the proportion of oxygen in the mixture (Fig. 4 left one), which also confirms the fact that the oxidation process is progressing.

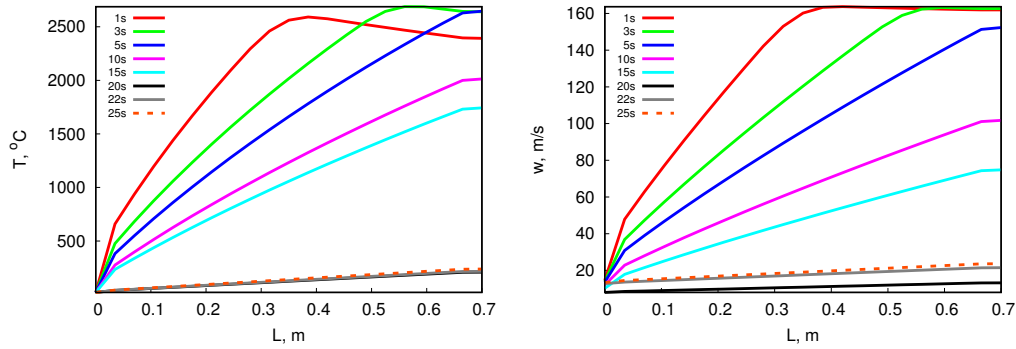


Figure 3: Distribution of the temperature (left) and exhaust gas velocity (right) along the rocket combustion chamber during the flight.

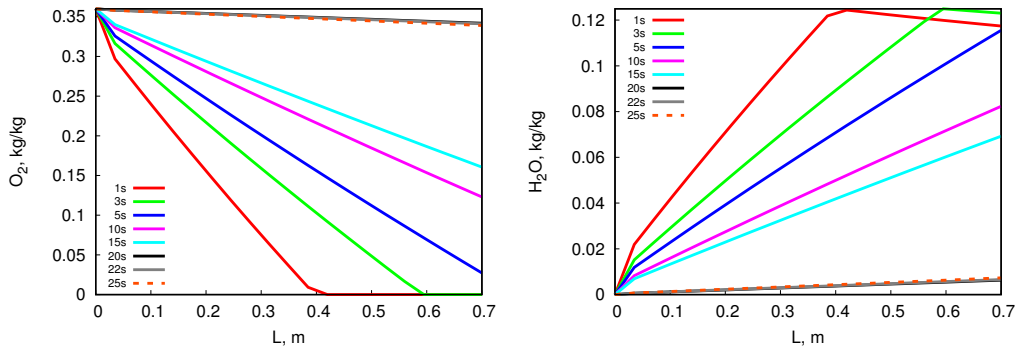


Figure 4: Distribution of the oxygen (left) and vapor (right) mass fractions along the rocket combustion chamber during the flight.

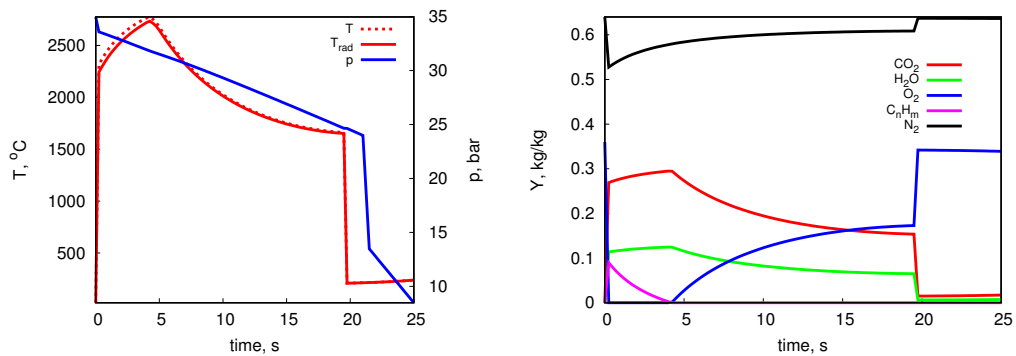


Figure 5: Distribution of the exhaust gas temperature and its pressure (left) and chemical compounds (right) at the outlet from rocket combustion chamber during the flight.

Fig. 5 shows the variation of selected parameters over time at the outlet of the combustion chamber. As can be seen, even the implementation of the simplest radiation model into the numerical code reduces the gas temperature by about 50°C (max. value 2750°C vs. 2700°C - Fig. 5 left graph). From the distribution of both the temperature (Fig. 5 left) and the content of individual gas components (Fig. 5 right), it can be deduced that the combustion process up to about 5s is sub-stoichiometric (excess of C_nH_m in the mixture), at 5s the oxidation is most intense (the temperature reaches the maximum what is associated with an approximately stoichiometric process), and after about 21s the fuel burns out while the fraction of O_2 in the mixture reaches the initial value.

The obtained results are consistent with those of other authors analyzing more complicated models. Uddanti et al. [16] created a 2D model of the rocket and using ANSYS FLUENT software for stationary calculations, obtained a maximum gas temperature of 3400°C at a maximum mixture velocity of about 200m/s. Moreover, these values appear in the exit area of the chamber. Obtained the average temperature at the outlet in their 2D CFD model reached value of 2230°C . In presented 1D quasi-stationary model this value is obtained at 8s of the flight. Our maximum value for temperature and velocity are about 2700°C and 160m/s, respectively, what gives the differences between obtained maximum values at the level of 20% for both parameters. In contrast, Rampazzo et al. [2] under similar assumptions showed a maximum temperature of 3100°C and a velocity of about 150m/s. In contrast to proposed model it gives the differences about 13% with respect to the predicted temperature and 7% according to calculated velocity. It is worth noticing that other authors used full 2D flow models taking into account both turbulence and chemical kinetics modeling at stationary flow conditions. The proposed 1D model is a quasi-stationary model that gives averaged parameters close to the results of other authors in a much shorter time. This is a clear advantage of using this type of approach.

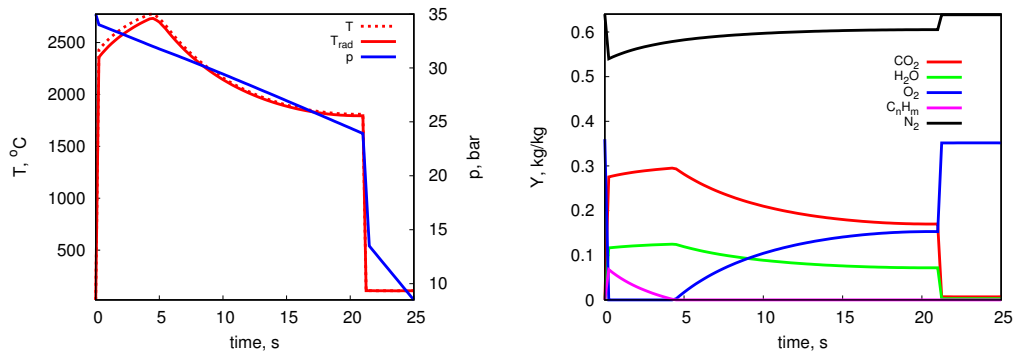


Figure 6: Distribution of the exhaust gas temperature and pressure (left) and chemical compounds (right) at the outlet from rocket combustion chamber during the flight.

In Fig. 6 simulation results of the oxidation process for the case with other fuel loading are presented. Compared to the previous test this one has over two times lower weight of core fuel. As it can be seen the combustion process in this case takes two seconds longer in comparison to the previous case (around 22s vs. 20s). The total mass of the rocket is lower, but it gives the smaller value of the velocity of exhaust gas (the average area is greater, so for the same mass flow rate the average velocity is lower - see Fig. 7) .

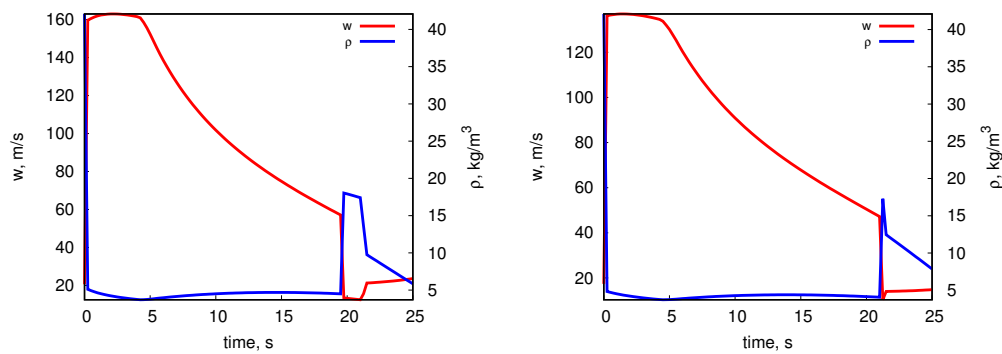


Figure 7: Distribution of the exhaust gas velocity and density at the outlet from rocket combustion chamber during the flight for heavier (left) and lighter (right) weight of the core fuel.

4 Conclusions

The presented numerical analysis shows that the adopted approach reduces the complexity, and thus reduces the computation time of numerical simulation of the combustion process in a hybrid rocket engine. The chosen method for prediction of distribution of exhaust gas parameters along the rocket combustion chamber during its flight can be a useful tool at the early stage of rocket design and avoids time-consuming, unstable and full-scale detailed phenomena analysis. In the future, to improve the accuracy of the model, it can be extended by more detailed kinetics of paraffin oxidation in nitrous oxide taking into account more sophisticated radiation model. To be more comparable and more practical, it should be also extended to include a nozzle module to obtain more practical parameters of the rocket, as thrust, and total and specific impulse.

References

- [1] Barato, F.: Review of Alternative Sustainable Fuels for Hybrid Rocket Propulsion. *Aerospace*, vol. 10, no. 643: (2023).
- [2] Rampazzo, A. & Barato, F.: Modeling and CFD Simulation of Regression Rate in Hybrid Rocket Motors. *Fire*, vol. 6, no. 3: 100 (2023).
- [3] Gieras, M. & Gorgeri A.: Numerical modelling of the hybrid rocket engine performance. *Propulsion and Power Research*, vol. 10, no. 1: (2021) pp. 15–22.
- [4] Heeg, F., Kilzer, L., Seitz, R. & Stoll, E.: Design and Test of a Student Hybrid Rocket Engine with an External Carbon Fiber Composite Structure. *Aerospace*, vol. 7, no. 5: 57 (2020).
- [5] Chelaru, T.V. & Mingireanu, F.: Hybrid rocket engine, theoretical model and experiment. *Acta Astronautica*, vol. 68: (2011) pp. 1891–1902.
- [6] Zolla, P.M., Migliorino, M.T., Bianchi, D., Nasuti, F., Pellegrini, R.C. & Cavallini, E.: A Computational Tool for the Design of Hybrid Rockets. *Aerotecnica Missili & Spazio*, vol. 100: (2021) pp. 253–262.
- [7] <https://aeronotes.weebly.com/hybrid-rocket-propulsion.html>
- [8] Puzyrewski, R. & Sawicki, J.: Fundamentals of fluid mechanics and hydraulics (in Polish). *PWN Scientific and Technical Publishers*, Warsaw (2000).
- [9] Pal, Y., Palateerdham S.K., Mahottamananda, S.N., Sivakumar, S. & Ingenito, A.: Combustion performance of hybrid rocket fuels loaded with MgB_2 and carbon black additives. *Propulsion and Power Research*, vol. 12, no. 2: (2023) pp. 212–226.

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- [10] Chen, S., Tang, Y., Zhang, W., Shen, R., Yu, H., Ye, Y. & DeLuca, L.T.: Innovative Methods to Enhance the Combustion Properties of Solid Fuels for Hybrid Rocket Propulsion. *Aerospace*, vol. 6, no. 4: 47 (2019).
 - [11] Kardaś, D. & Szyborski, J.: Development of a novel model for emptying of a self-pressurising nitrous oxide tank. *Journal of Physics: Conference Series*, vol. 1781: 012001 (2020).
 - [12] Szyborski, J. & Kardaś, D.: Novel model for emptying of a self-pressurised nitrous oxide tank. *Archives of Thermodynamics*, vol. 43, no. 3: (2022) pp. 141–173.
 - [13] Kalinowski, E.: Heat transfer and heat exchangers (in Polish). *Oficyna Wydawnicza Politechniki Wrocławskiej*, Wrocław, Poland (1995).
 - [14] McBride, B.J., Zehe, M.J. & Gordon, S.: NASA Glenn Coefficients for Calculating Thermodynamic Properties of Individual Species. *National Aeronautics and Space Administration Glenn Research Center Cleveland, Ohio 44135*, NASA/TP—2002-211556 (2002).
 - [15] Kosma Z.: Numerical methods for engineering applications. *Radom Technical University* (1999).
 - [16] Uddanti, N. S. & Crispin, Y.: CFD Modeling of a Hybrid Rocket Using a Generalized One-dimensional Model of the Flame Temperature. *AIAA Propulsion and Energy* (2019).