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HYDRODYNAMIC INSTABILITIES IN SPHERICAL COUETTE FLOW WITH FREE SURFACE EFFECTS: EXPERIMENTS AND SIMULATIONS

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Abstract

This paper presents an experimental and numerical study of hydrodynamic instabilities of an incompressible viscous fluid contained between two concentric spheres whose the inner one rotating ($\Omega_1 \neq 0$) while the outer is at rest ($\Omega_2 = 0$). The flow system, totally or partially filled through the aspect ratio $\Gamma = H/d$ of the fluid is defined by clearance ratio $\delta = d/R_1$ in small gap configuration ($\delta = 0.11$). The main aim is to investigate the effect of the aspect ratio on the instabilities in flow field system with free surface using water as working fluid. Experiments are performed for different aspect ratio values and Laser photometric technique is used for visualization. Our observations are compared to the flow system totally filled. The effect of free surface on the onset of axisymmetric Taylor vortices instability and oscillating vortices are examined in the spherical Couette flow annulus. A good agreement was observed between experimental and numerical results showing that the wavy mode is accelerated for weak aspect ratio.

Keywords: Spherical Couette flow, free surface flow, aspect ratio, instabilities, CFD.

1 Introduction

Significant progress has been made in the fields related to the study of the appearance of instability phenomena and bifurcations in the laminar-turbulent transition regime. The objective is to resolve the problems encountered during the control of certain industrial processes and to explain their mechanisms and properties. Several theoretical and experimental studies have been devoted to the case of flows confined in a spherical annular gap. The aim is to examine the different types of centrifugal instabilities subjected to the influence of friction forces by viscosity and inertia. Indeed, the movement of an incompressible viscous fluid between two spheres is very sensitive to the rotation velocity. When the speed increases, the structure of the movement and the regime, which governs it, are modified and are characterized by a phenomenon of successive symmetry breaking. The transition of this regime becomes complex and ends with the onset of chaos followed by the establishment of fully developed turbulence.

Moreover, the phenomenological modes can be further complicated when we consider this flow with two superimposed immiscible fluids and in the presence of a free surface which introduces significant modifications acting on the patterns form.

The study of the movement of a viscous fluid in such a configuration in a fully filled vertical position called "Spherical Couette flow SCF" is of paramount importance aiming at a better knowledge of the fundamental mechanisms to explain the process of laminar-turbulent transition and the development of turbulence. This system has a wide variety of phenomena of successive appearance of hydrodynamic instabilities in a transition regime [1]. Indeed, many studies have been carried out in this field in order to understand the nature of the phenomena of instabilities related to the transition to turbulence by attempting to give an explanation to the pattern of instabilities or bifurcations of the flow that s turns out to be very complex in this case.

The flow between two coaxial rotating spheres was theoretically studied in 1961 by Iu. K. Bratukhin [2] whose inner sphere is rotating ($\Omega_1 \neq 0$) and the outer sphere is fixed ($\Omega_2 = 0$). Assuming the Stokes flow is the basic laminar flow. An approximate result of linear stability in the flow driven by the rotating internal sphere was obtained for a radial clearance $\delta = 1$ (corresponding to a geometric factor $\eta = 0.5$).

He found a critical Reynolds number $Re_c \approx 100$ corresponding to the onset of instability, which is in good agreement with the energy theory ($Re_c = 90$). B. R. Munson and D. D. Joseph [3, 4] theoretically studied the laminar motion of an incompressible viscous fluid between two spheres that rotate at different angular speeds. Using an analytical solution, they were able to determine the distribution law of the mean velocity field for low values of the Reynolds number. The study carried out for the largest numbers of Re was approached by the Galerkin method. The structure of the flow as a function of the Reynolds number and the necessary torque imposed on the sphere for a given rotation has been determined.

The theoretical genesis of the bifurcation of the asymmetric state has been analyzed by C. K. Mamun and L. S. Tuckerman [5]. They adapted a pseudo-spectral formulation to calculate stable and unstable states using Newton's method and linear stability analysis using Arnoldi's method. It was found that the asymmetric branch results from a forked bifurcation and its stabilization occurs by a subsequent subcritical Hopf bifurcation. They used a complete computational approach by dominating the scenario according to the state of flow with a single vortex (1-vortex) with other asymmetric states corresponding to an average annular gap.

Using visualization measurements which is accompanied by a spectral study, K. Nakabayashi and Y. Tsuchida [6] studied the laminar-turbulent transition of the flow between two concentric spheres of which only the inner sphere is rotating for two values of the radial clearance: $\delta = 0.06$ and $\delta = 0.024$.

According to the previous analysis, there are several works that are devoted to the study of the effect of the width of the annular gap δ, namely, thin, medium or wide, on the phenomenon of the appearance of instabilities. In practice, this parameter significantly affects the size and number of Taylor cells as a function of the critical Reynolds number. Some authors have estimated that Taylor cells would not exist in wide spherical annular gaps for a radial clearance $\delta > 0.24$. This assertion is contradicted by experimental and numerical work which has shown that Taylor cells can exist for a large radial clearance according to G. N. Klebuthin [7], Yu. N. Balyeav [8], M. Liu et al. [9], K. Nakabayashi [10] and others. The experimental study by G. N. Klebuthin [7] confirmed that Taylor cells do not appear in a transition regime beyond the critical value of radial clearance $\delta_c = 0.44$. On the other hand, Yu. N. Balyeav [8] found numerically that this value can reach $\delta_c = 0.483$. This result is consistent with observations made in the cylindrical T-C system which show that Taylor cells no longer exist beyond $\delta_c > 0.53$.

The effect of eccentricity on the appearance and development of Taylor cells at different instability regimes has been studied numerically by P. Bar-Yoseph et al. [11] for different ray ratios. Different eccentricities were analyzed on the evolution of Taylor vortices and the transition between flow modes. They observed the peculiar phenomenon of hysteresis and when the eccentricity increases this process disappears. The comparison between the flow modes was carried out by numerical simulation which seems to be in good agreement with the experimental results. The effect of natural or forced convection on the transition of the instability phenomena of a flow contained in a spherical annular gap has been examined by many authors: E. H. Bishop et al. [12], J. A. Scanlan et al. [13], N. Weber et al. [14] and S. N. Singh and J. M. Elliot [15]. More recent numerical studies on thermal convection in a spherical annular gap have also been carried out in particular by N. Scurtu et al. [16], Feldman and Colonius [17] and A. Akbar Dehghan and M. Khoshab [18], V. Travnikov et al. [19] and others.

In this paper, the influence of the aspect ratio on the onset of Taylor vortices first instability and wavy mode of spherical Couette flow has been examined. A good agreement was observed between experimental and numerical results showing that the wavy mode is accelerated for a weak aspect ratio.

2 Experimental method and conditions

Fig. 1 shows the experimental set-up which consists of two rotating concentric spheres characterized by the inner and outer radii R_1 and R_2 respectively. The system is intended for the use of several interchangeable spheres and designed to facilitate quick assembly and disassembly operations. The outer sphere is 100 mm in the inner diameter, and made of Plexiglas to help flow visualization. The inner sphere is 90 mm in the outer diameter, and made of a Plexiglas. The radial gap is d = 5mm so that the radius ratio of the spherical Couette system is $\delta = d/R_1 = 0.11$. The total maximum aspect ratio is $\Gamma = 20$.

The geometry and a diagram of the spherical annulus is shown in Fig. 1. An incompressible fluid with constant kinematic viscosity fills the gap between the two concentric spheres. The outer sphere is fixed and the inner one rotates with angular velocity Ω_1. The working fluid is water of density ρ = 999.6 kg/m^3 and kinematic viscosity $\nu = 1.01 \cdot 10^{-6}$ Pa·s at T = 20 °C with kalliroscope AQ1000 in a small amount (2 vol %). The visualization of regimes and structures associated to movement is performed by means of photometric technique based on optical reflection and transmission. The appearance of critical regimes is highlighted by laser, which allows a local observation of the cellular structure of the flow.

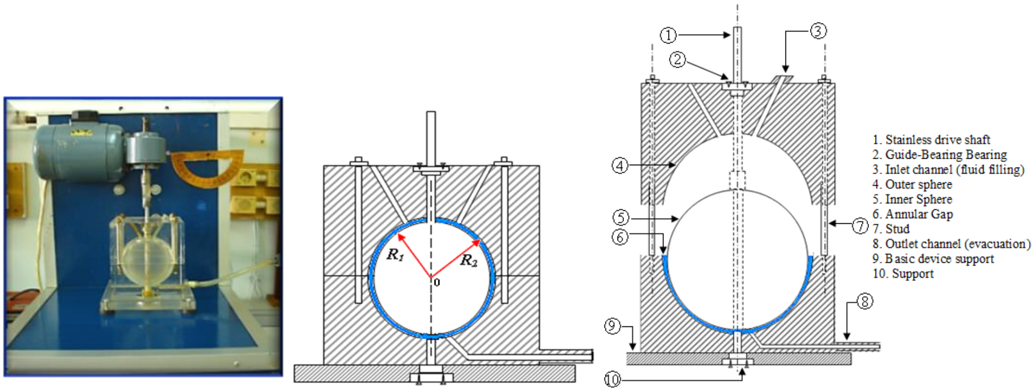


Figure 1. Experimental device and schematic diagram of spherical Couette flow

The different regimes and the transition thresholds of spherical Taylor-Couette flow depend on the rotation rate of the inner sphere. The inner sphere velocity was kept constant with a relative accuracy $\Delta\Omega_1/\Omega_1 \leq 1$ % where $\Delta\Omega_1$ is the velocity increasing. The velocity rate is controlled with a numerical tachymeter of type DT2236. The temperature is measured by an electronic captor (-50 to +150°C) with an error lower than 1 %.

For low inner sphere velocity, the flow regime exhibits two large cells that travel from each pole to the equator. The flow behaviour is determined by the Taylor number. By increasing the Taylor number, we observe Taylor vortices TVF near the equator.

The Taylor number is defined by:

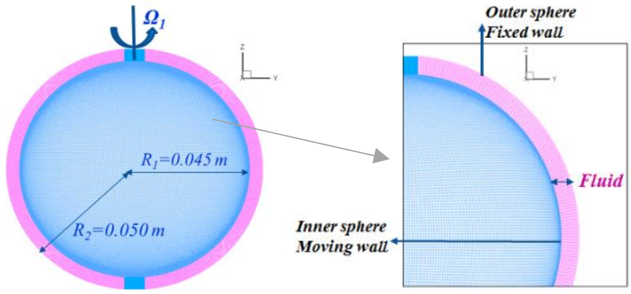
$$\mathrm{Ta} = \mathrm{Re}\sqrt{\frac{d}{R_1}}$$

As the radius of inner sphere R_1 is chosen as characteristic length and $\Omega_1 R_1$ as characteristic velocity, the Reynolds number is, thus, $\mathrm{Re} = \frac{\Omega_1 R_1^2}{\nu}$.

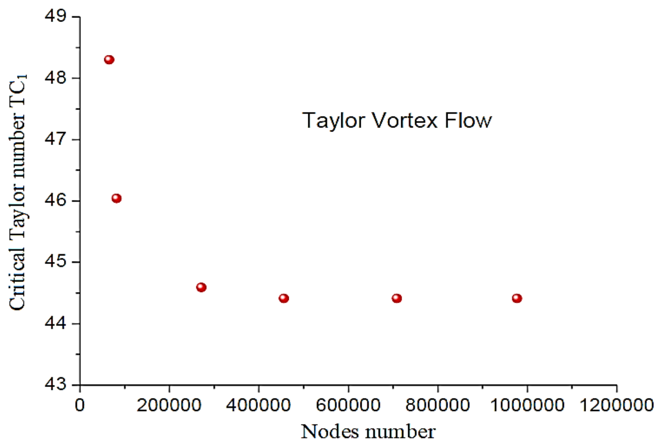
Where, Re, d, R_1, Ω_1 and ν are the Reynolds number, gap size, radius of the inner sphere, angular velocity of the inner sphere and kinematic viscosity of the fluid. There are five critical Taylor numbers during the flow transition. Increasing the angular velocity modify the flow transition regimes and may alter even the flow structure itself. Since only the inner sphere rotates, the observed transition flow regimes are Taylor vortex flow, spiral mode, spiral wavy mode, wavy vortex flow, chaos and turbulence.

In this study, we highlight the influence of the free surface on the produced instabilities for different aspect ratios. To better understand the specific phenomenology of the flow, we have one interfacial dynamic, i. e. surface instability and one volume dynamic (Taylor vortices). The experiments were carried out for different values of the aspect ratio $\Gamma = H/d$ and the observations were carried out using a photometric technique and by laser. This experimental work led to the results, which allow an analysis of the studied cases. Numerical results were obtained using the Fluent simulation code, which employs the finite volume method for equation discretization. The mesh was created using GAMBIT software. A structured hexahedral mesh, consisting of approximately 907 200 cells, was selected. The "PRESTO" model was chosen for pressure discretization, while the "Third-order MUSCL" scheme was applied to the momentum equations. For pressure-velocity coupling, the "PISO" model was selected due to its versatility. The free surface flow in spherical Couette flow was analyzed using the Volume of Fluid (VOF) method. Convergence was achieved when all normalized residuals were below 10^{-6}. The boundary conditions are as follows:

Inner sphere	**Outer sphere**
$(\Omega_1 \neq 0)$: $r = R_1$: $V_r = V_\theta = 0$, $V_\phi = \Omega_1 R_1 \sin\theta$	$(\Omega_2 = 0)$: $r = R_2$: $V_r = V_\theta = V_\phi = 0$



(a)



(b)

Figure 2. (a) Grid distribution of the computational domain (hexahedral mesh)
(b) Mesh independence test

3 Results

We observed a transition to spherical Couette flow when the Taylor number was increased. This occurred by rotating only the inner sphere. In that case, the spherical Couette flow remains stable until the onset of the first formation of the Taylor vortex flow (TVF) instability that occurs in the gap. The transition behavior strongly depends on the angular velocity.

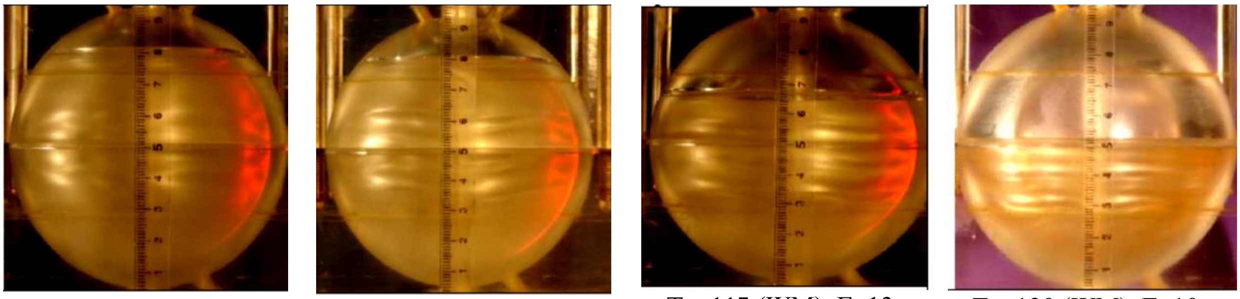
By increasing angular velocity, many other flow modes have been observed in addition to the Taylor vortices and spherical Couette flow such as spiral mode, spiral mode with oscillating Taylor vortices, spiral wavy mode and wavy Taylor vortices.

The experiments were carried out by varying the aspect ratio of the flow in order to exhibit the promising interaction of the free surface on the various transition regimes observed. These results are compared to those obtained in the fully filled system.

In order to study the effect of the aspect ratio on the conditions of appearance of the instabilities considered, one proceeded to fill the flow system H. For each aspect ratio Γ fixed, one has varied the Taylor number for each case to determine the instabilities encountered. In the present work, we limit our study only to the first instability TVF and wavy Taylor vortices (wavy mode).

The Taylor number is increased stepwise by increasing the angular velocity of the inner sphere. In spherical gap, the hydrodynamic instabilities are developed in the equatorial plane where the centrifugal force on the flow is highest.

In addition, we observed the interaction between the flow and free surface that leads to the generation of surface cell near the interfacial plane liquid-air as shown in Fig. 3 for Ta = 115 (WM) and $\Gamma = 13$. This surface cell, that appears early before the onset of Taylor vortices, has not been observed before to our best knowledge in SCF system.



Ta=45 (TVF), Γ=16 Ta=195 (WM), Γ=16 Ta=115 (WM), Γ=13 Ta=120 (WM), Γ=10

Figure 3. Visualization of spherical Couette flow system with free surface for different aspect ratio *Γ*

The first Taylor vortices (TVF) outcomes in the formation of a pair of toroidal vortices around the equator in each hemisphere. This regime is stationary; the position of Taylor vortices can be expressed by stationary sink, and source corresponding to the critical Taylor number Ta_{c1}. The first instability is stationary axisymmetric vortices for a very low angular velocity at a Taylor number of Ta_{c1} (TVF) = 44. Beyond this value of the Taylor number, the flow becomes more disturbed and the observed Taylor vortices are more oscillating. In fact, there are very emerging disturbances that directly affect the free surface. For example, the surface is distorted at Ta_{c5} (WM) =100.5 and Ta_{c5} (WM) = 99.56 for, $\Gamma = 13$ and 10 respectively. Some experimentally observed flows and structures on the front view of the outer sphere such as Taylor vortices and wavy regime (axial direction) for different aspect ratios are shown in Fig. 3.

Fig. 4 shows the evolution of critical Taylor number versus the aspect ratio in the range $10 \le \Gamma \le 19$. We can see that the instability of the Taylor vortex flow (TVF) corresponding to Ta_{c1} is not affected by the aspect ratio in the studied range. Its value is almost the same than in the case of a completely filled system (Ta_{c1} = 43.09). On the other hand, we found that the regime most affected by the free surface is the wavy mode. The curve obviously shows a larger jump from the wavy mode for aspect ratio $13 < \Gamma \le 19$ and $10 \le \Gamma \le 13$. The aspect ratio of decrease in critical Taylor numbers is greater than 46%.

We found that the Wavy mode instability is strongly accelerated in the flow, and for this reason, we performed a numerical simulation to validate our experimental results.

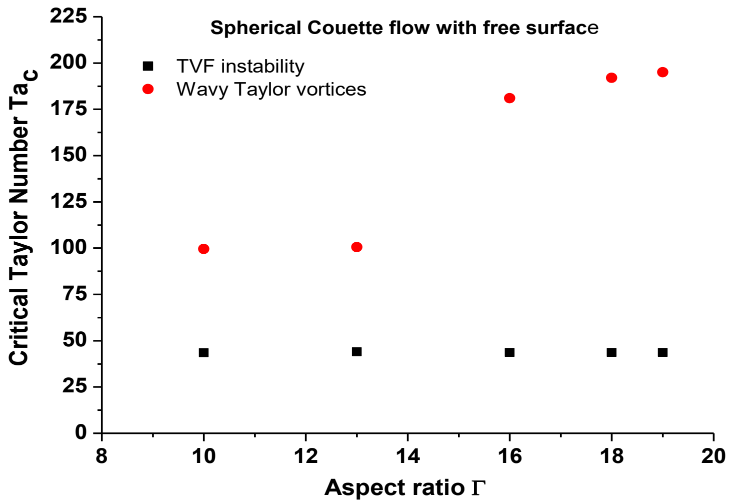
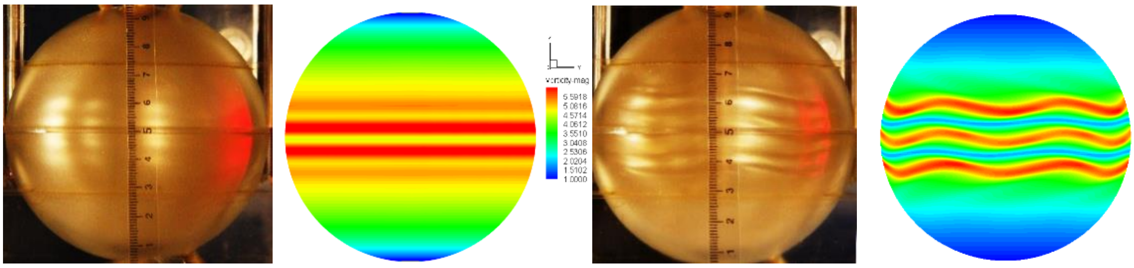


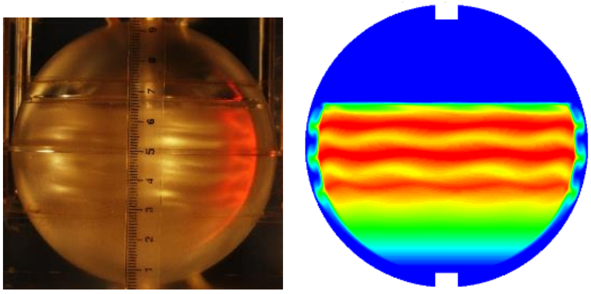
Figure 4. Evolution of critical Taylor number versus the aspect ratio Γ

Three-dimensional numerical simulations are performed using the finite volume method. The VOF model is used to set up and solve the free-surface problem in spherical Taylor-Couette flow. We have numerically reproduced the experimentally observed Taylor vortices at $Ta_{c1} = 44.4$ and wavy mode at $Ta_{c5} = 224$ and $Ta_{c5} = 112$ (Fig. 5). The simulation confirms our experimental measurements that a surface cell exists and wavy mode flow is accelerated for low aspect ratio. The numerical results of the flow pattern and critical Taylor numbers were analyzed and compared to the experimental results, which are in good agreements. In particular, we confirm by the streamlines created in the meridian plane (r, θ) the existence of one surface cell as interfacial instability that is clearly formed (Fig. 6). We find that this interfacial vortex takes place asymmetrically with respect to the equator.



TVF, Ta_{c1} = 43 [20] TVF, Ta_{c1} = 44.4 WM Ta_{c5} = 218[20] WM Ta_{c5} = 224

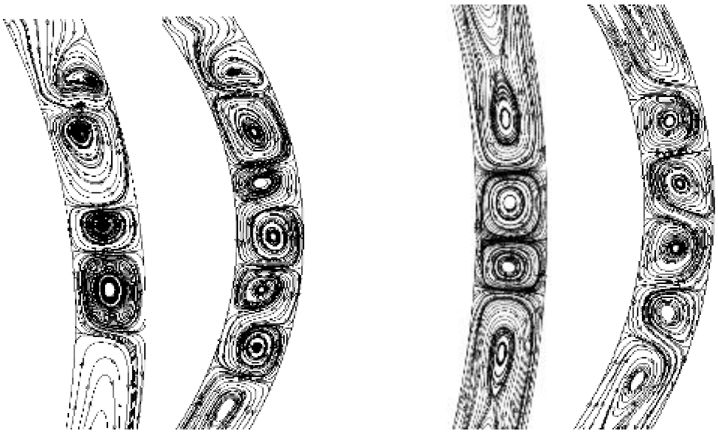
(a) Γ=20 (system without free surface)



Ta_{c5} =115 Ta_{c5} =112

(b) Γ=13, Wavy Mode (system with free surface)

Figure 5. Experimental and numerical comparison of hydrodynamic instabilities in SCF (Taylor vortex flow and Wavy Mode instability)



Γ = 13, Ta_{c1} = 44.4 Γ = 20, Ta_{c5} = 224

Figure 6. Representation of the streamlines in the meridian plane (r, θ): Comparison of the aspect ratio effects in the equatorial region for Γ = 13 and Γ = 20.

Fig. 7 illustrates the variation of the dynamic pressure near of the equator under the effect of the aspect ratio Γ and for Taylor number value Ta = 15 and Ta_{c5} = 224.

For a weak Ta = 15, we notice that the dynamic pressure which represents the effects of the free surface Γ = 13 seems 10 times greater than the one found in the case of the flow totally filled (flow system without free surface). As a result, the flow undergoes a significant and rapid increase in pressure in the annular gap in comparison with that obtained at Γ = 20 which is almost constant (unlike the situation of azimuthal waves the values of the dynamic pressure appear a lot bigger).

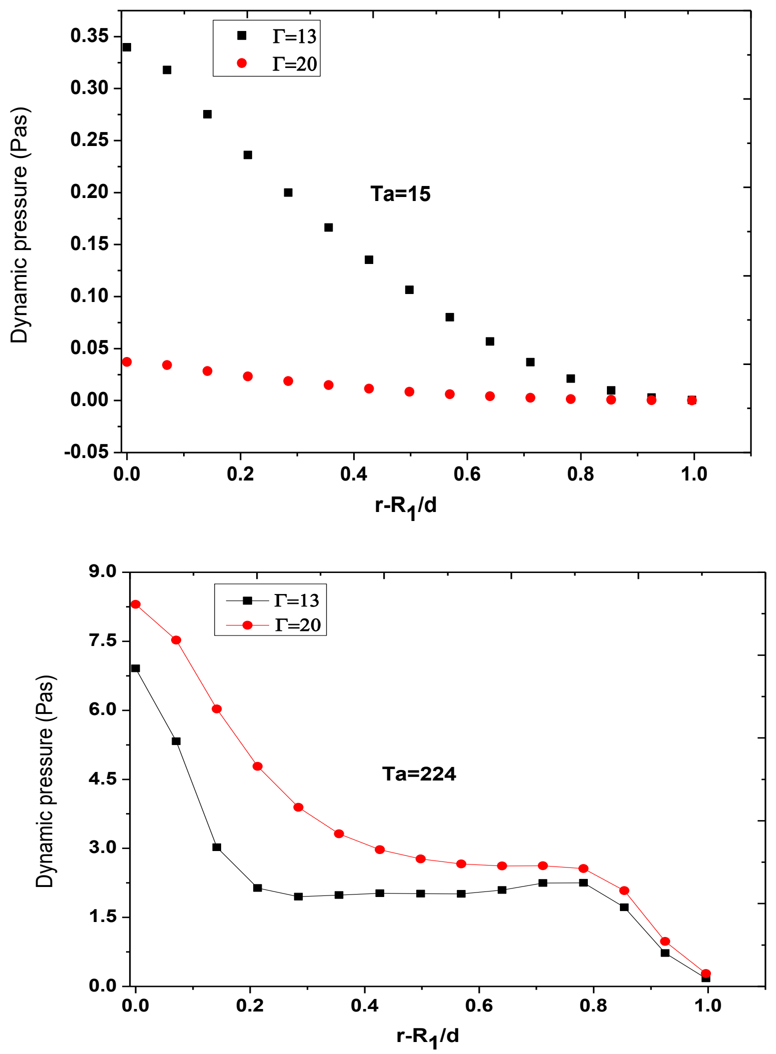


Figure 7: Variation of the dynamic pressure at the equator for different values of Ta

4 Conclusion

The flow field with and without free surface between two concentric spheres is studied experimentally and numerically. The outer sphere is fixed while the inner sphere rotates. The critical Taylor number values of the appearance of the Taylor vortex flow and wavy mode for different aspect ratios are determined. The onset of wavy regime is mostly affected by the free surface causing an acceleration on which the aspect ratio of decrease is more than 46% in comparison with case without free surface.

The numerical investigation confirms the existence of a surface cell, which was observed in the experiments. We found numerically that the oscillating wavy mode is accelerated for weak aspect ratio.

Dynamic pressure along the spherical gap is calculated for low and high Taylor numbers.

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